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Ion passage over the solar wind termination shock under conservation of particle invariants in view of Voyager-2 observations

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Abstract. We study the plasma passage over astrophysical MHD shocks with frozen-in magnetic fields in arbitrary inclinations with respect to the bulk plasma motion. As a function of the compression ratio at the shock, we aim at the prediction of ion plasma properties downstream of the shock, especially the resulting downstream temperatures, pressures and pressure anisotropies as function of the upstream magnetic tilt angle. Using dynamical invariants governing the ion motions, we derive the independent reactions of the ion velocity components parallel and perpendicular to the local magnetic field at the shock passage. This allows us to determine not only the associated downstream ion velocities, but the ion distribution function and its velocity moments like pressures and temperatures. We find pronounced increases of the downstream ion temperatures with respect to corresponding upstream values. The down-to-up temperature ratios thereby strongly depend on the upstream magnetic tilt angle attaining maximum values in case of a quasi-perpendicular shock. We also obtain fire-hose-unstable temperature anisotropies with values $T_\perp/T_\parallel \ll 1$ at small tilt angles (quasi-parallel shock) and mirror-unstable values of anisotropies $T_\perp/T_\parallel \gg 1$ at tilt angles near $\pi/2$ (quasi-perpendicular shock).

1 Introduction

Astrophysical shocks are highly important dynamical structures in space plasmas, not only because they transport and dissipate over large dimensions huge amounts of energies that mostly originate in stars and stellar winds, but also because they serve as particle accelerators and X-ray radiators. To describe these latter phenomena in a satisfactorily reliable manner, one needs well developed and appropriately formulated systems of magnetohydrodynamic conservation equations that help to determine the plasma properties downstream of the shock. It is, however, recognized for quite some while now that a complete description of the ion passage over an MHD shock with the help of a standard set of MHD shock relations (see Baumjohann and Treumann, 1996; Gombosi, 1998; Diver, 2001) is not possible, since the amount of entropy generation at the shock passage in these relations remain unspecified. This becomes clearly evident for anisotropic plasmas for which the MHD shock relations define a system of under-determined equations that do not allow for the derivation of an unequivocal set of downstream plasma properties (see Erkaev et al., 2000; Vogl et al., 2003; Liu et al., 2007; Génot, 2008; Fahr and Siewert, 2008; Siewert and Fahr, 2008).

In the following part of this paper we shall revisit the basis of these earlier results and shall thereby demonstrate that when using an appropriate coordinate system for the description of the shock and when using dynamical ion invariants will then help to overcome most of the earlier problems in giving unequivocal representations of the downstream plasma properties.

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2 A revised setting of the scene of MHD jump conditions

2.1 The tangential mass flow - an overlooked parameter

As emphasized throughout the literature, the generally used anisotropic system of MHD jump conditions is underdetermined, with only 6 equations available to determine 7 downstream plasma parameters (see e.g. Gombosi, 1998; Erkaev et al., 2000; Diver, 2001; Vogl et al., 2003; Siewert and Fahr, 2008). Considerable effort has been put into this field over the last decade (e.g. by Erkaev et al., 2000; Fahr and Siewert, 2006; Liu et al., 2007; Lucek et al., 2007; Siewert and Fahr, 2008; Génot, 2008; Treumann, 2009; Fahr and Siew-

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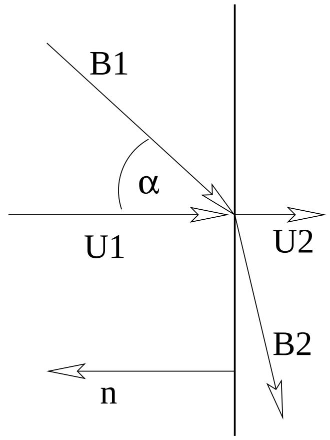

Fig. 1. The geometrical configuration of the solar wind termination shock.

ert, 2010) ranging from isolating one downstream parameter that needs to be determined by other independent methods to more ambitious theoretical attempts to extract additional information from careful kinetic modeling. In this section, we introduce a revised approach to this issue, based exclusively on MHD, but helping to overcome the above mentioned problem.

We now repeat a few selected results of the theory of MHD jump conditions. Shock waves appear as discontinuities in the solutions of the MHD equations, meaning that MHD alone is unable to describe the physical processes resulting in the jump of MHD parameters, leaving only conservation equations to describe the discontinuity. One of these equations is based on the more general conservation of the mass flow in general plasma physics:

$$\rho_1 \boldsymbol{U}_1 = \rho_2 \boldsymbol{U}_2. \tag{1}$$

This is a vector-valued equation, which in principle results in three individual scalar conservation equations. Now, in most theoretical studies, the shock is studied in a special rest frame, characterised by $\boldsymbol{U} \times \boldsymbol{n} = 0$, where $\boldsymbol{n}$ is the normal vector defining the orientation of the shock surface. In other words, Eq. (1) boils down to one conserved component $U_n = \boldsymbol{U} \cdot \boldsymbol{n}$ of the mass flow, i.e.

$$\rho_1 U_{n1} = \rho_2 U_{n2}. \tag{2}$$

This commonly found simplification holds the potential for misunderstandings. Even though the upstream tangential velocity component of the plasma flow vanishes (i.e. $U_{t1} = \boldsymbol{U} \times \boldsymbol{n} = 0$), the downstream velocity component typically is different from zero (see e.g. (Erkaev et al., 2000; Vogl et al., 2003)). This may be understood in the context of a reference frame which is at motion with U_{t1} perpendicular to the magnetic field. In a rest frame at rest at the shock, and using Eq. (1), the tangential velocity flow must be conserved as well, i.e.

$$\rho_1 U_{t1} = \rho_2 U_{t2}. \tag{3}$$

This equation is only valid in the shock rest frame, though; in a more general rest frame moving with U^*, we obtain the more general formulation

$$\rho_1 (U^*_{t1} + U^*) = \rho_2 U_{t2}. \tag{4}$$

Therefore, any tangential mass flow which vanishes on one side of the shock, but not on the other one is a clear hint for a moving (but still inertial) frame of reference. (It should be noted that Eq. (3) can also be proven more rigorously using a finite shock layer, the limit to infinite thinness and some mathematical arguments, but since this proof is not relevant to this study, we shall leave it out at this point.)

At this point, we have to remind the reader that the new tangential mass flow conservation remains consistent with the classical MHD jump conditions. If, for example, we take the frozen-in field condition (Eq. (2.2) of Erkaev et al., 2000), we obtain a downstream tangential velocity of

$$U_{t2} = \frac{U_{n2} B_{t2} - U_{n1} B_{t1} + U_{t1} B_{n1}}{B_{n2}}. \tag{5}$$

For $U_{t1} = 0$, we obtain $B_{t2} = s B_{t1}$, and $U_{n2} B_{t2} = U_{n1} B_{t1}$, i.e. $U_{t2} = 0$, as required by the new relation. If, on the other hand, $U_{t1} \neq 0$, we obtain

$$U_{t2} = \left(1 + \frac{U_{n2} B_{t2} - U_{n1} B_{t1}}{U_{t1} B_{n2}}\right) U_{t1}, \tag{6}$$

i.e.

$$s = \left(1 + \frac{U_{n2} B_{t2} - U_{n1} B_{t1}}{U_{t1} B_{n2}}\right)^{-1}. \tag{7}$$

From Eq. (2.4) of Erkaev et al. (2000), it additionally follows that

$$s = \frac{1}{1 - \beta_{\parallel 2} + \beta_{\perp 2}}. \tag{8}$$

All together this means that for the case we are considering in this paper the so-called "kinematic approximation" applies where magnetic forces do not influence the plasma dynamics.

In the following studies, we shall consider a rest frame that is explicitly at rest with the point-like shock transition. This allows us to compliment the MHD jump conditions with the new Eq. (3), a condition which is notably absent from the vast majority of the shock literature, where a specific reference frame for the jump conditions is not always explicitly defined.

This choice allows us to introduce a seventh equation into the system of jump conditions, allowing in principle a unique solution of the 7 free downstream parameters. This result hints that the system of Rankine-Hugoniot jump conditions may not be as underdetermined as quoted throughout literature, but instead be perfectly closed, with the missing equation being systematically discarded. We shall study this improved set of jump conditions in a future publication.

2.2 The solar wind termination shock

In this publication, we study the solar wind termination shock in the upwind region of the heliosphere, where the solar wind is facing the interstellar gas inflow. In this system, the supersonic solar wind meets the shock surface along the shock normal ($\boldsymbol{U}\parallel\boldsymbol{n}$, i.e. $U_{t1}=0$ in the rest frame of the shock, see Fig. 1), where Eq. (3) is perfectly valid without any additional modifications. In this case, it follows from Eq. (2) that

$$\frac{\rho_2}{\rho_1}=\frac{U_{n1}}{U_{n2}}=s, \tag{9}$$

which is the classical first step in solving the jump conditions. Now, however, it follows from Eq. (3) that, similarly,

$$\frac{\rho_2}{\rho_1}=\frac{U_{t1}}{U_{t2}}=s, \tag{10}$$

meaning that for $U_{t1}=0$, we automatically obtain $U_{t2}=0$. Furthermore, it follows from the frozen-in field condition (Eq. (2.2) of Erkaev et al., 2000) and $U_{t2}=0$ that

$$B_{t2}=\frac{U_{n1}}{U_{n2}}B_{t1}=s\,B_{t1}. \tag{11}$$

This result is notably different (and simpler) from Erkaev et al. (2000) and results from the specific choice of the reference frame. For this above explained scenario, we now derive kinetic properties for the plasma passage over the shock.

3 Selected properties of the kinetic model

3.1 Introduction to ion velocity invariants and velocity coordinate transformations

It is well known from basic plasma physics that in absence of stochastic processes like collisions or wave-particle interactions ions moving along magnetic fields with a field magnitude gradient along the field line behave, as if they have to conserve a dynamic quantity called their magnetic moment $\Gamma=(m/2)v_{\perp}^2/B$ (see e.g. Chen, 1984; Baumjohann and Treumann, 1996). In addition it is, however, interesting to note that this quantity Γ also plays the role of an invariant, if ions are co-convected with a plasma bulk into the motion of which a magnetic field is frozen in. If in the direction of the bulk flow the frozen-in magnetic field magnitude decreases, as for instance is the case for the Archimedian spiral field in the inner heliosphere (see Parker (1958), or Forsyth et al. (2002)), then it can be shown that ions, while being co-convected with the plasma bulk, also in this case have to conserve their magnetic moment Γ (see Fahr, 2007; Fahr and Siewert, 2008; Fahr and Siewert, 2010). The only restriction hereby is that typical periods τ_B of the field magnitude changes be large compared to the ion gyroperiods τ_g.

We now summarise and expand the basic arguments given in Fahr and Siewert (2008); Fahr and Siewert (2010), i.e. under which conditions the magnetic moment is conserved at MHD shock waves. As we have shown in this reference, it can be proven by solving the equation of motion for individual ions in the plasma bulk system that ions do in fact conserve their magnetic moment at their passage over the shock structure, if their gyration period τ_g is short compared to the passage time $\tau_p=D_{TS}/U_1\gg\tau_g$. We now generalise this proof and demonstrate that the same result also applies to systems where $\tau_g\gg\tau_p$ or $\tau_g\simeq\tau_p$.

For the case of large gyroperiods ($\tau_g\gg\tau_p$), only at a very small passage along one gyroperiod, an electric induction force operates that changes the velocity. This force follows from Faradays induction law,

$$\int_0^{\Delta\varphi}\boldsymbol{E}_{\mathrm{ind}}\cdot d\boldsymbol{s}=-\int_O\frac{\partial\boldsymbol{B}}{\partial t}\cdot d\boldsymbol{O}\ , \tag{12}$$

where we integrate over the surface O covered by the line joining the gyrational center and the ion (i.e. only that fraction of a full gyration period where the force is active). Assuming that all vectors are located in one plane and selecting $d\boldsymbol{s}=r_g d\varphi$, we obtain

$$\Delta\varphi r_g E_{\mathrm{ind}}=-\frac{\Delta\varphi}{2\pi}\pi r_g^2\frac{\partial B}{\partial t}\ . \tag{13}$$

Therefore, the energy transfer induced by the action of this force is given by

$$\frac{m}{2}\frac{dv_{\perp}^2}{dt}=eE_{\mathrm{ind}}v_{\perp}=-\frac{e}{2}r_g v_{\perp}\frac{\partial B}{\partial t}\ , \tag{14}$$

which again may be transformed into (Fahr and Siewert, 2008; Fahr and Siewert, 2010)

$$\frac{v_{\perp}^2}{B}=const. \tag{15}$$

For the more complicated case of $\tau_g\simeq\tau_p$, the properties in Eq. (12) become time-dependent, resulting in

$$\int_t^{t+\Delta t}\boldsymbol{E}_{\mathrm{ind}}(t)\cdot d\boldsymbol{s}(t)=-\int_O(\Delta t)\frac{\partial\boldsymbol{B}}{\partial t}\cdot d\boldsymbol{O} \tag{16}$$

for any given time t at which the force is acting. This then results in

$$\Omega_g(t)\Delta t\cdot r_g(t)E_{\mathrm{ind}}(t)=-\frac{\Omega_g(t)\Delta t}{2\pi}\pi r_g^2(t)\frac{\partial B}{\partial t}\ , \tag{17}$$

where $\Omega_g=eB/mc$ is the gyrofrequency. This results in

$$\frac{dv_{\perp}}{dt}=-\frac{c}{2}\frac{v_{\perp}}{B}\frac{\partial B}{\partial t} \tag{18}$$

and finally in

$$\frac{v_{\perp}}{\sqrt{B}}=const.\ , \tag{19}$$

which is proportional to the square root of the magnetic moment.

Even though these calculations prove that the magnetic moment is conserved in the vast majority of shocked systems, there are more complicated contributions which might invalidate these arguments. For example, there may be shock-reflected particles which in velocity space are located outside the loss cone of the shock-generated magnetic mirror configuration. Such reflections can only occur for suprathermal upstream ions with thermal velocities much higher than the bulk velocity U_1 (see e.g. Terasawa, 1979). Anyway, most of these ions cannot escape upstream, but are again swept downstream with the frozen in magnetic field. It is, however, very interesting to note that even in the study by Terasawa (1979), it was already found that independent on the relevant timescales (i.e. whether $\tau_g \le \tau_p$ or $\tau_g \ge \tau_p$, these reflected ions behave adiabatically, i.e. conserve their magnetic moment.

Another contribution which has not been studied in detail yet is the effect of the magnetic field strongly changing its orientation during one gyration period, i.e. $\boldsymbol{B}(t) \nparallel \boldsymbol{B}_2$. This might result in a complication of Stokes theorem, especially when the concept of the enclosed surface over one gyration breaks down. For situations where magnetic field reorientation is sufficiently slow, the magnetic moment however is conserved in first order approximation.

In addition connected with differential motions of ions parallel to frozen-in fields, in case of bulk velocity gradients parallel to the field, there appears a quantity that can serve as a second particle invariant $\Gamma_\parallel = v_\parallel B/\rho$ (Siewert and Fahr, 2008; Fahr and Siewert, 2008), which is associated with the second plasma invariant in the CGL-theory (Chew et al., 1956).

These ion invariants can profitably be used to also describe the change of the dynamical properties of ions at their passage over the solar wind termination shock where both an abrupt decrease of the solar wind bulk velocity and an increase of ion densities and of frozen-in magnetic field magnitudes occur.

3.2 A new representation of the kinetic invariants

We now derive a new formulation of the kinetic invariants by introducing the magnetic field angle $\alpha = \angle(\boldsymbol{n}, \boldsymbol{B})$ (and, again, selecting $\theta = \angle(\boldsymbol{n}, \boldsymbol{U}) = 0$), where $\boldsymbol{n}$ is the shock normal. The single-particle formulation of the invariants are given in the following form (Siewert and Fahr, 2008):

$$\frac{v_\perp^2}{B} = \frac{v_{\perp 1}^2}{B_1} = \frac{v_{\perp 2}^2}{B_2} = const. \tag{20}$$

and

$$\frac{v_\parallel B}{\rho} = \frac{v_{\parallel 1} B_1}{\rho_1} = \frac{v_{\parallel 2} B_2}{\rho_2} = const. \tag{21}$$

Introducing the angle α and using Eqs. (9 – 11) and $B_n = const.$ (see Erkaev et al., 2000), it is possible to eliminate the magnetic field magnitude ratio,

$$\frac{B_2}{B_1} = \sqrt{\frac{B_{n2}^2 + B_{t2}^2}{B_{n1}^2 + B_{t1}^2}} = \sqrt{\frac{B_{n1}^2 + s B_{t1}^2}{B_{n1}^2 + B_{t1}^2}}. \tag{22}$$

This expression may now be parameterized in terms of the upstream magnetic field angle α, by introducing $B_n = B\cos\alpha$ and $B_t = B\sin\alpha$, finally leading to the result

$$\frac{B_2}{B_1} = \sqrt{\cos^2\alpha + s^2 \sin^2\alpha}. \tag{23}$$

This is an improvement over the earlier results obtained by Siewert and Fahr (2008), since it eliminates the previously unknown ratio B_2/B_1 with a function depending only on upstream parameters and the MHD compression ratio s.

Using this equation, the upstream-to-downstream relations for the individual velocity components transform into

$$v_{\perp 2}^2 = v_{\perp 1}^2 \frac{B_2}{B_1} = v_{\perp 1}^2 \sqrt{\cos^2\alpha + s^2 \sin^2\alpha} \tag{24}$$

and

$$v_{\parallel 2}^2 = v_{\parallel 1}^2 \left(\frac{B_1 \rho_2}{B_2 \rho_1}\right)^2 = v_{\parallel 1}^2 \frac{s^2}{\cos^2\alpha + s^2 \sin^2\alpha}. \tag{25}$$

Unifying the above results, we may derive the total kinetic energy gain suffered by an individual ion at the shock passage with the upstream velocity $\boldsymbol{v}_1$ and an upstream ion pitch angle $\beta_1 = \angle(\boldsymbol{B}, \boldsymbol{v})$:

$$\begin{aligned} v_2^2 =& v_1^2 \Bigg(\sin^2\beta_1 \sqrt{\cos^2\alpha + s^2 \sin^2\alpha} \\ &+ \cos^2\beta_1 \frac{s^2}{\cos^2\alpha + s^2\sin^2\alpha} \Bigg) \\ =& v_1^2 C(\alpha, \beta), \end{aligned} \tag{26}$$

where we have introduced the notation

$$\begin{aligned} C(\alpha,\beta) =& \sin^2\beta_1 \sqrt{\cos^2\alpha + s^2\sin^2\alpha} \\ &+ \cos^2\beta_1 \frac{s^2}{\cos^2\alpha + s^2 \sin^2\alpha}. \end{aligned} \tag{27}$$

Using $v_\parallel = v\cos\beta$ and $v_\perp = v\sin\beta$, as well as Eqs. (24) and (25), we obtain the downstream ion pitch angle by

$$\mathrm{ctg}^2\beta_2 = \mathrm{ctg}^2\beta_1 \cdot \frac{s^2}{(\cos^2\alpha + s^2\sin^2\alpha)^{3/2}}. \tag{28}$$

It is obvious that this equation does automatically introduce an anisotropy of the distribution function and a pressure anisotropy on the downstream side.

4 Downstream pressures and temperatures

4.1 Isotropic distribution functions

Using the new parameterisation of the shock properties with the upstream magnetic field tilt angle α, we now briefly demonstrate that the resulting upstream-to-downstream relations between the corresponding pressure components remain unchanged from what was derived by Fahr and Lay (2000); Siewert and Fahr (2008) as long as isotropic functions are treated. Beginning with the assumptions made by Fahr and Lay (2000), we also want to make the additional assumption that due to rapid pitch-angle scattering by Alfvén-wave turbulence normally occuring without energy changes of the ions. Therefore, one may expect to obtain pitch-angle isotropic distribution functions both on the upstream side, and also on the downstream side after some relaxation processes have taken place over a flow distance of $D \simeq U_2 \tau_{\mu\mu}$.

In this publication, we are only interested in the pressure (or, equivalently, the temperature) of the plasma, which is given on the downstream side by

$$p_2 = \frac{4\pi}{3}\frac{m}{2}\int_0^{\pi}\int_0^{\infty}\sin\beta_2\, v_2^4\, f_2(v_2)\, dv_2\, d\beta_2. \tag{29}$$

The integral can be simplified by applying Liouville's theorem (similar to the approach by Fahr and Lay (2000)), which states that the differential phasespace flux is conserved, i.e. $\Psi_1(v_1) = \Psi_2(v_2)$. Expressing the gyro-averaged phase space flow by $\Psi = 2\pi U f(\boldsymbol{v})\, v^2 \sin\beta\, d\beta dv$, we obtain

$$\begin{aligned} &U_2 f_2(\boldsymbol{v}_2)\, v_2^2 \sin\beta_2\, d\beta_2 dv_2 \\ &\qquad = U_1 f_1(\boldsymbol{v}_1)\, v_1^2 \sin\beta_1\, d\beta_1 dv_1\,. \end{aligned} \tag{30}$$

Multiplying this with v_2^2/U_2, we obtain

$$\begin{aligned} &f_2(\boldsymbol{v}_2)\, v_2^4 \sin\beta_2\, d\beta_2 dv_2 \\ &\qquad = \frac{U_1}{U_2} f_1(\boldsymbol{v}_1)\, v_1^2 v_2^2 \sin\beta_1\, d\beta_1 dv_1\,. \end{aligned} \tag{31}$$

Using this equation and Eq. (26), we can transform Eq. (29) into

$$\begin{aligned} p_2 &= \frac{4\pi}{3}\frac{m}{2} s \int_0^{\pi} \sin\beta_1 \int_0^{\infty} v_1^4 C(\alpha,\beta_1)\, f_1(v_1)\, d\beta_1 dv_1 \\ &= p_1\, s \int_0^{\pi} \sin\beta_1 C(\alpha,\beta_1)\, d\beta_1 dv_1. \end{aligned} \tag{32}$$

Evaluating the remaining integral finally yields

$$p_2 = p_1\, s\, \left(\frac{2}{3}\sqrt{\cos^2\alpha + s^2\sin^2\alpha} + \frac{1}{3}\frac{s^2}{\cos^2\alpha + s^2\sin^2\alpha}\right). \tag{33}$$

The large bracket, derived from the integral over C, may be interpreted as the weighted average over the parallel and perpendicular velocity contributions, where the perpendicular pressure weights twice as much as the parallel pressure, since it is related to gyrational motion, i.e. two cartesian coordinate directions. Typical results for the corresponding temperature increase $T_2/T_1 = p_2/p_1 \cdot \rho_1/\rho_2$ over the full range of possible of magnetic field angles α and typical compression ratios s are presented in Fig. 2.

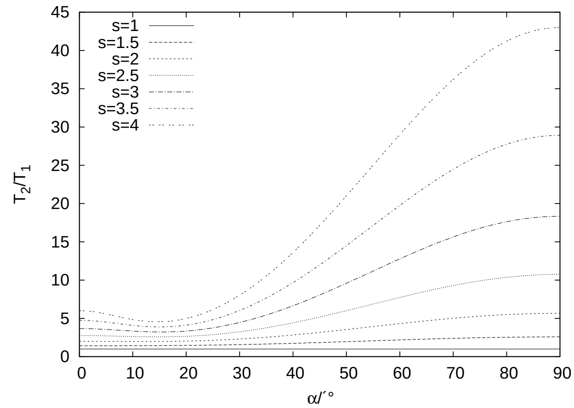

Fig. 2. Temperature gains from the upstream to the downstream side after pitchangle isotropisation as a function of the magnetic field angle α and the compression ratio s.

4.2 Anisotropic distribution functions

Now, we study the kinetic properties of the downstream plasma in a configuration before rapid pitchangle scattering at ALFénic turbulences occurs, i.e. where the downstream system is explicitly anisotropic (with a distribution function denoted by $f = f^{\mathrm{aniso}}$). The parallel and perpendicular pressures are then defined by

$$\begin{aligned} p_\perp &= \frac{4\pi}{3}\frac{m}{2}\int f^{\mathrm{aniso}}(\boldsymbol{v})\, v_\perp^2\, d^3v \\ &= \frac{4\pi}{3}\frac{m}{2}\int f^{\mathrm{aniso}}(\boldsymbol{v})\, v^2 \sin^2\beta\, d^3v \end{aligned} \tag{34}$$

and

$$\begin{aligned} p_\parallel &= \frac{4\pi}{3}\frac{m}{2}\int f^{\mathrm{aniso}}(\boldsymbol{v})\, v_\parallel^2\, d^3v \\ &= \frac{4\pi}{3}\frac{m}{2}\int f^{\mathrm{aniso}}(\boldsymbol{v})\, v^2 \cos^2\beta\, d^3v. \end{aligned} \tag{35}$$

To derive a function $p_{i,2} = g(p_{i,1})$, i.e. an expression similar to Eq. (33), we start with Eqs. (24) and (25) and obtain with

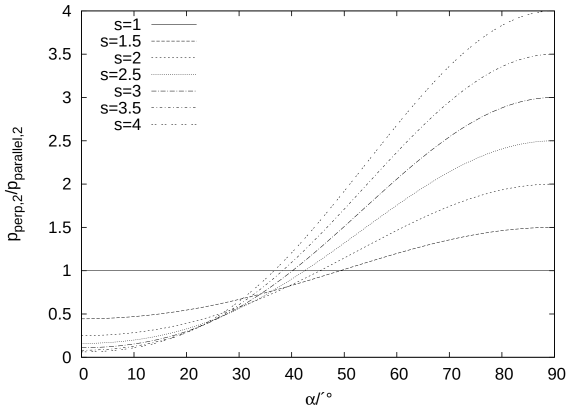

Fig. 3. Downstream pressure anisotropies as a function of the magnetic field angle α and the MHD compression ratio s before pitchangle isotropisation.

the Liouville relation

$$\begin{aligned} &2\pi v_2^2 U_2 f_2^{\mathrm{aniso}} dv_2 v_2^2 \cos^2\beta_2 \sin\beta_2 d\beta_2 \\ =&2\pi v_1^2 U_1 f_1^{\mathrm{aniso}} dv_1 v_1^2 \cos^2\beta_1 \frac{v_{\|2}^2}{v_{\|1}^2} \sin\beta_1 d\beta_1 \\ =&2\pi v_1^2 U_1 f_1^{\mathrm{aniso}} dv_1 v_1^2 \cos^2\beta_1 \\ &\left(\frac{s^2}{\cos^2\alpha + s^2\sin^2\alpha}\right) \sin\beta_1 d\beta_1 \end{aligned} \tag{36}$$

and

$$\begin{aligned} &2\pi v_2^2 U_2 f_2^{\mathrm{aniso}} dv_2 v_2^2 \sin^2\beta_2 \sin\beta_2 d\beta_2 \\ =&2\pi v_1^2 U_1 f_1^{\mathrm{aniso}} dv_1 v_1^2 \sin^2\beta_1 \frac{v_{\perp 2}^2}{v_{\perp 1}^2} \sin\beta_1 d\beta_1 . \\ =&2\pi v_1^2 U_1 f_1^{\mathrm{aniso}} dv_1 v_1^2 \sin^2\beta_1 \\ &\sqrt{\cos^2\alpha + s^2\sin^2\alpha} \sin\beta_1 d\beta_1 . \end{aligned} \tag{37}$$

Integrating these above relations over velocity space then leads to the two relations

$$\begin{aligned} p_{\|,2} =& \frac{4\pi}{3}\frac{m}{2}\frac{U_1}{U_2}\frac{s^2}{\cos^2\alpha + s^2\sin^2\alpha} \\ &\int\int v_1^4 f_1^{\mathrm{aniso}} dv_1 \cos^2\beta_1 \sin\beta_1 d\beta_1 \\ =& p_{\|,1}\frac{s^3}{\cos^2\alpha + s^2\sin^2\alpha} \end{aligned} \tag{38}$$

and

$$\begin{aligned} p_{\perp,2} =& \frac{4\pi}{3}\frac{m}{2}\frac{U_1}{U_2}\sqrt{\cos^2\alpha + s^2\sin^2\alpha} \\ &\int\int v_1^4 f_1^{\mathrm{aniso}} dv_1 \sin^2\beta_1 \sin\beta_1 d\beta_1 \\ =& p_{\perp,1} s\sqrt{\cos^2\alpha + s^2\sin^2\alpha} . \end{aligned} \tag{39}$$

These above results deliver the following downstream pressure anisotropy:

$$\begin{aligned} \frac{p_{\perp,2}}{p_{\|,2}} =& \frac{\sqrt{\cos^2\alpha + s^2\sin^2\alpha}}{\frac{s^2}{\cos^2\alpha + s^2\sin^2\alpha}} \\ =& \frac{p_{\perp,1}}{p_{\|,1}}\frac{(\cos^2\alpha + s^2\sin^2\alpha)^{3/2}}{s^2} . \end{aligned} \tag{40}$$

A graphical representation of this equation is given in Fig. 3.

Since this generalized result is based on the same physical concepts as those used by Siewert and Fahr (2008) for special tilt angles, it should be equivalent to their earlier results for $\alpha = 0$ and $\alpha = \pi/2$. For a parallel shock ($\alpha = 0$), Eq. (40) leads to their earlier result,

$$\frac{p_{\perp,2}}{p_{\|,2}} = \frac{1}{s^2}\frac{p_{\perp,1}}{p_{\|,1}} , \tag{41}$$

and the perpendicular shock ($\alpha = \pi/2$), we obtain

$$\frac{p_{\perp,2}}{p_{\|,2}} = s\frac{p_{\perp,1}}{p_{\|,1}} , \tag{42}$$

which is, again, equivalent to the results by Siewert and Fahr (2008). In addition to this, Siewert and Fahr (2008) also derived a more general expression for arbitrary angles α,

$$\frac{p_{\perp,2}}{p_{\|,2}} = \frac{1}{s^2}\left(\frac{B_2}{B_1}\right)^3 \frac{p_{\perp,1}}{p_{\|,1}} . \tag{43}$$

This result is similarly compatible with our new results, where the explicit upstream magnetic field angle α is taken into account (i.e. Eqs. (11) and (40)).

It must be noted that this simplification would not have been possible without explicitly taking the conservation of the tangential mass flow into account (Eq. (3)), without which the tangential magnetic field would be considerably more complicated (see Erkaev et al. (2000), Eq. 5.1).

4.3 The entropy gain across the shock

With the obtained results, we now are provided with the ingredients to derive the thermal entropy jump across the shock as a function of α. According to Erkaev et al. (2000), the entropy on the upstream and downstream sides is given by the expression

$$S = \frac{k}{2}\ln\frac{p_\perp^2 p_\|}{\rho^5} \tag{44}$$

Using this expression and our Eqs. (39) and (38), we can thus derive the downstream entropy,

$$\begin{aligned} S_2 =& \frac{k}{2}\ln\frac{p_{\perp 2}^2 p_{\|2}}{\rho_2^5} \\ =& \frac{k}{2}\ln\frac{p_{\perp,1}^2 s^2(\cos^2\alpha + s^2\sin^2\alpha)\frac{s^3}{\cos^2\alpha + s^2\sin^2\alpha} p_{\|,1}}{s^5\rho_1^5} \end{aligned} \tag{45}$$

which simplifies to

$$S_2 = \frac{k}{2} \ln \frac{s^2 p_{\perp,1}^2 s^3 p_{\parallel,1}}{s^5 \rho_1^5} = S_1. \tag{46}$$

This means that the thermal entropies obviously do not depend on the tilt angle α and have not been increased at the shock passage, if the conservation of dynamical particle invariants is fulfilled. This is true even though thermal heating does occur inside the system. Nevertheless the entropy has been increased due to an increase of the kinetic entropy and further will be increased by stochastic scattering processes connected with the excitement of waves of the fire-hose mode or of the mirror-mode (see Fahr and Siewert, 2007).

In this paper we do not aim at treating this situation quantitatively, but shall take a look on that situation which results after the initially anisotropic ion distribution due to pitchangle scattering processes has been rearranged into a pitchangle-isotropic distribution $f_2(v_2) = \langle f_2(v_2, \beta_2) \rangle_\beta$. In Fig. 2, we have thus shown the isotropic downstream temperature T_2 calculated with this isotropized function $f_2(v_2) = \langle f_2(v_2, \beta_2) \rangle_\beta$ according to Eq. (33). Using this expression, it is now possible to derive the entropy gain due to pitchangle isotropisation after first building up an anisotropy, which is obtained by generalising Eq. (44) to

$$S = \frac{k}{2} \ln \frac{P_\perp^2 P_\parallel}{\rho^5} = \frac{k}{2} \ln \frac{P^3}{\rho^5} \ . \tag{47}$$

On the downstream side, this equation transforms into

$$\begin{aligned} S_2 =& \frac{k}{2} \ln \frac{p_1^3}{s^2 \rho_1^5} \\ &+ \frac{k}{2} \ln \left(\frac{2}{3} \sqrt{\cos^2\alpha + s^2 \sin^2\alpha} + \frac{1}{3} \frac{s^2}{\cos^2\alpha + s^2 \sin^2\alpha} \right)^3 \\ =& S_1 + \frac{k}{2} \ln \frac{1}{27 s^2} \\ &+ \frac{k}{2} \ln \left(2\sqrt{\cos^2\alpha + s^2 \sin^2\alpha} + \frac{s^2}{\cos^2\alpha + s^2 \sin^2\alpha} \right)^3 . \end{aligned} \tag{48}$$

This result clearly demonstrates that only after the pitchangle anisotropy has been removed by stochastic pitchangle scattering processes one obtains a physical entropy increase. This expression, which depends both on the tilt angle α and the compression ratio s, is displayed in graphical form in Fig. 4, which clearly demonstrates that the entropy gain is largest for parallel shocks, average for perpendicular shocks and minimal for inclined shocks at around $\alpha \simeq 40°$, where only a small initial downstream pressure anisotropy is generated, which in turn nicely correlates with the absence of an entropy increase.

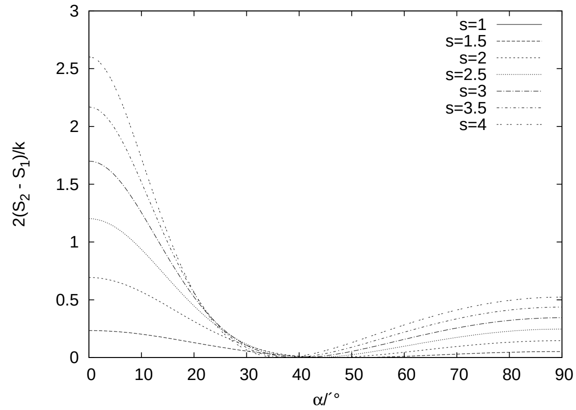

Fig. 4. Normalised entropy gain across the shock as a function of the magnetic field angle α and the MHD compression ratio s after pitchangle isotropisation.

5 Conclusions

We have shown that in an appropriate description of the shock we can find unequivocal solutions for the downstream velocity moments of the plasma. Our description is especially suited to describe the solar wind termination shock in ecliptic regions of the upwind hemisphere of the solar system, since tangential upstream bulk velocities can be neglected here. Since this is just the type of shock that recently has been crossed by the two NASA spaceprobes Voyager-1/-2 there exist interesting possibilities to compare our results with in-situ space plasma measurements.

With the use of the newly derived dynamical ion invariants we first transform ion velocity components from upstream to downstream of the shock and show that immediately after shock passage an anisotropic ion distribution is established which can be characterized by anisotropic temperatures $T_{2,\parallel}$ and $T_{2,\perp}$. In Fig. 3, we have shown that the resulting temperature anisotropies $A_{2,\parallel}^{\perp} = T_{2,\perp}/T_{2,\parallel} = s p_{\perp,2}/p_{\perp,1}$ strongly depend on the upstream magnetic tilt angle α. Small values of $A_{2,\parallel}^{\perp} \ll 1$ result for small values of α (i.e. quasi-parallel shocks), while large values $A_{2,\parallel}^{\perp} \geq 1$ result for tilt values of $\alpha \simeq \pi/2$ (i.e. quasi-perpendicular shock). As evident from Fig. 3, these types of anisotropies furthermore are the stronger pronounced the larger is the compression ratio s.

The resulting downstream temperature anisotropies can be checked with respect to associated unstable ion velocity distributions driving, dependent in strength on the prevailing magnetic $\beta_{\perp,\parallel}$ - values, either fire-hose instabilities in case of quasi-parallel shocks or mirror instabilities in case of quasi-perpendicular shocks (see discussions in Fahr and Siewert, 2007; Génot, 2008; Fahr and Siewert, 2009; Lazar and Poedts, 2009; Siewert and Fahr, 2009). Interestingly enough, the conversion of an isotropic upstream into the associated anisotropic downstream distribution does not imply a thermal entropy increase, which is exclusively related to the following reisotropisation on the far downstream side due to pitchangle scattering processes.

It also becomes evident that with Eq. (33) and the result shown in Fig. 2, we can give a fairly satisfying explanation of the downstream solar wind proton temperatures which has been measured by Voyager-2 at the cross-over of the termination shock in September 2007 (see Richardson et al., 2008). According to Fig. 2, we would expect to see temperature increases by factors of 10 till 12, while Richardson et al. (2008) show measured values fluctuating between 10 to 15. In contrast, the classical MHD shock relations instead would have predicted a temperature increase by a factor of about 10^2 which by far is not reflected in the data.

More important, however, is the recognition of the strongly pronounced sensitivity of the downstream plasma properties on the upstream magnetic tilt angle α. Most of the time a magnetic tilt angle $\alpha \simeq \pi/2$ may be realized at the upwind near-ecliptic termination shock, however, it is interesting to keep in mind that during passages of magnetic sector structures determined by the interplanetary magnetic current sheath one will have changes of the tilt angle from $\alpha = +\pi/2$ over $\alpha = 0$ to $\alpha = -\pi/2$ within about 2 days (see e.g. Fahr et al., 2008; Scherer and Fahr, 2009). It is thus recommended to study the plasma passage over the shock under time-variable upstream magnetic tilt angles $\alpha(t)$. We shall look into this interesting problem in an upcoming paper.

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