

Observations of anomalous transport of energetic electrons in the heliosphere

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Abstract. We study the propagation of energetic particles, accelerated by interplanetary shock waves, upstream of the shock, considering the possibility of having anomalous transport. The theoretical treatment of the anomalous transport is developed via the propagator formalism, considering that in the case of a superdiffusive transport the propagator has power law tails with slope $2 < \mu < 3$, while for a ballistic transport it is approximately constant for small displacements and has a spike for $r \simeq vt$. Analyzing a data set from Ulysses spacecraft in the period between July 1992 and November 1993, we find that the time profile of electrons accelerated at a traveling planar shock is a power law rather than an exponential decay, implying superdiffusive motion. We also show that the propagator formalism allows to describe the scatter-free propagation, possible for electrons accelerated in impulsive solar events and observed by WIND. These results indicate that the propagation of energetic particles in the turbulent environment of the solar wind can be anomalous, in agreement with the results of recent numerical simulations, and promise to have application to the propagation of particles throughout the heliosphere.

1 Introduction

One of the most elusive subjects in space physics is the propagation of particles in the interplanetary medium. Indeed, many phenomena can play a role in this process, as, for example, the solar activity, the level of turbulence in the heliosphere, the energy of particles, and the presence of discontinuities. In the last 30 years normal (Brownian-like) diffusion has been largely considered in connection with diffusive shock acceleration (DSA) (Jokipii, 1966; Bell, 1978; Bland-

ford and Ostriker, 1978; Lee and Fisk, 1982) that explains many experimental evidences as the cosmic rays power law spectra (Hillas, 2005). This theory is a first order Fermi acceleration mechanism where particles are accelerated during shock encounters, which allow them to reach very high energies being diffuse back to the shock owing to magnetic irregularities (e.g., Lee, 2005). An efficient acceleration is favoured by small mean free paths. However, analyzing data from WIND, IMP-8 and Ulysses spacecraft, many authors have pointed out the possibility of an anomalous diffusion. Indeed, the transport of solar energetic particles (SEP) appears to be ballistic (scatter-free) up to $\simeq 1$ AU (Reames, 1999; Zhang et al., 2003; Lin, 2005). In addition, since either in fluid and plasma experiments (Zaslavsky, 2002) and in numerical simulations (Giacalone and Jokipii, 1999; Qin, Matthaeus and Bieber, 2002; Zimbardo, Pommois and Veltri, 2006; Pommois, Zimbardo and Veltri, 2007) anomalous and normal transport regimes have been observed, we cannot exclude the possibility that the propagation of particles in space can deviate from a Brownian-like motion. We introduce the propagator formalism for anomalous transport, then, we investigate the propagation properties of energetic particles in the heliosphere and we show that the fluxes of energetic electrons accelerated at corotating interaction region (CIR) shocks, observed by Ulysses spacecraft at $\simeq 5$ AU, indicate a superdiffusive transport for this species. By studying the electron fluxes detected by WIND at $\simeq 1$ AU, we confirm the scatter-free propagation proposed by Lin (2005), although a larger class of propagators, corresponding to a power law distribution of displacements, can be involved.

2 Theoretical framework

Anomalous transport is characterized by a mean square displacement growing as a power law in time, i.e. $\langle x^2(t) \rangle \propto t^\alpha$. It can be both slower ($\alpha < 1$) and faster ($\alpha > 1$) with respect



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to Gaussian diffusion ($\alpha = 1$). The faster regime, $\alpha > 1$ is called superdiffusion and various theoretical approaches have been developed in recent years in order to describe it. For instance, it is characterized by random walks with very long jumps correlated in space and time (Lèvy random walks), and by a probability of making a jump of length r in a time t that is a power law, $\psi(r, t) = A|r|^{-\mu}\delta(t - r/v)$ (Shlesinger, West and Klafter, 1987; Klafter, Shlesinger and Zumofen, 1996), this indicates that very long jumps have small but non-negligible probability. With such a jump probability, superdiffusion is obtained for $2 < \mu < 3$, and ballistic transport for $1 < \mu < 2$; in addition, normal diffusion is recovered for $\mu > 3$ since the second order moment of the jump length distribution is finite, even if the statistics remains non Gaussian. Further, normal Brownian-like motion has a Gaussian jump length distribution with a finite second order moment. In Sokolov et al. (2002) and Zaslavsky (2002) the anomalous transport is treated via the formalism of fractional derivatives equations giving rise to a generalization of the standard diffusion equation, $\partial^\alpha/\partial t^\alpha P(\mathbf{r}, t) = k_\alpha \nabla^2 P(\mathbf{r}, t)$, where $\partial^\alpha/\partial t^\alpha$ represents the Riemann–Liouville fractional derivative and k_α is the fractional diffusion coefficient with dimension $[L^2/T^\alpha]$, being L and T space and time dimensions, respectively. However, an alternative method of studying anomalous diffusion is based on the propagator $P(\mathbf{r}, t)$, which describes the probability of finding a particle in (\mathbf{r}, t) , if it was injected at the origin. The propagator associated to a diffusive process has a Gaussian shape and for a one dimensional case is

$$P(r, t) \simeq a_1 t^{-1/2} \exp\left[-a_2 r^2/t\right], \quad (1)$$

where a_1 and a_2 are constants. The propagator associated to anomalous transport has been obtained by Zumofen and Klafter (1993). For a superdiffusive regime the expression of the propagator in Fourier–Laplace space can be easily inverted for the small and the large r expansion. In the latter case $P(r, t)$ is a power law in space

$$P(r, t) \simeq bt/r^\mu, \quad (2)$$

where $2 < \mu < 3$ and b is a constant; in addition the propagator goes to zero when $r > vt$, being v the particle velocity. Zumofen and Klafter (1993) also show that in the superdiffusive regime the mean square displacement grows in time as $\langle r^2(t) \rangle \propto t^\alpha = t^{4-\mu}$. On the other hand, when transport is ballistic, i.e. scatter-free, the power law exponent has to be $1 < \mu < 2$, but the propagator form is different from that in Eq. (2) (Zumofen and Klafter, 1993). For $\mu = 3/2$ an explicit inversion is possible and the process is well described by

$$P(r, t) \simeq \frac{c_1}{\pi(v^2 t^2 - r^2)^{1/2}}. \quad (3)$$

This expression holds for $r < vt$, while for $r > vt$ it goes to zero. At a fixed time, this propagator is nearly constant for small values of r , while it tends to diverge as r approaches vt ,

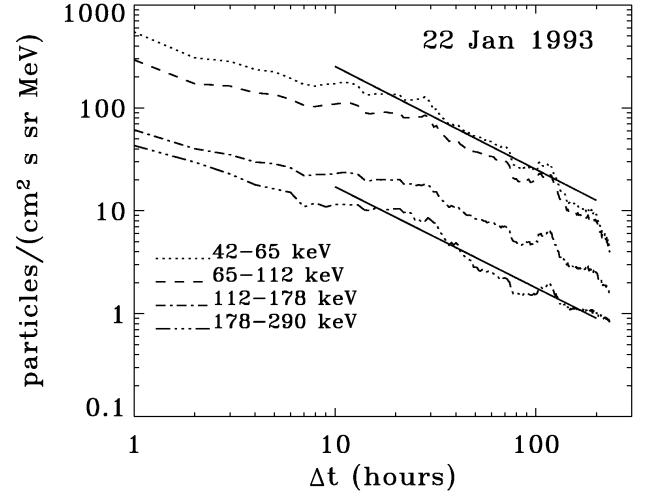


Fig. 1. Electron fluxes in log-log axes for the shock crossing observed by Ulysses at $\simeq 5$ AU. Solid lines show the best fit (for clarity not reported for all energy channels). Energies as indicated. From Perri and Zimbardo (2007), copyright Am. Astr. Soc.

because the bulk of particles tends to propagate together. The δ -like spike for $r = vt$ is similar to the δ -function propagator for strictly scatter-free transport, while the power law tail for $r < vt$ corresponds to those particles which undergo some scattering.

3 Observations of electron anomalous transport

We analyse particle time profiles from spacecraft data, that reflect the propagation properties from the source to the observer, as well as the source evolution. We consider events in which electrons are accelerated by interplanetary shock waves, as CIR shocks, and by impulsive events in the solar atmosphere, and we show that the former correspond to superdiffusive transport, while the latter to ballistic propagation.

Considering a large planar shock, that is a good first approximation for CIR shocks that have a radius of curvature comparable with the heliocentric distance, the particle distribution function is related to the propagator via the equation (Perri and Zimbardo, 2007, 2008)

$$f(x, E, t) = \int P(x - x', t - t') f_{\text{sh}}(x', E, t') dx' dt', \quad (4)$$

where $f_{\text{sh}}(x', E, t') = f_0(E) \delta(x' - V_{\text{sh}} t')$, being $f_0(E)$ the distribution function of particles of energy E emitted at the shock position and V_{sh} the upstream shock speed in the solar wind rest frame. For the DSA theory, at some distance upstream of the shock, where the magnetic turbulence level is nearly constant (Lee, 2005), the particle profile in the solar wind rest frame has a simple exponential shape (Fisk and

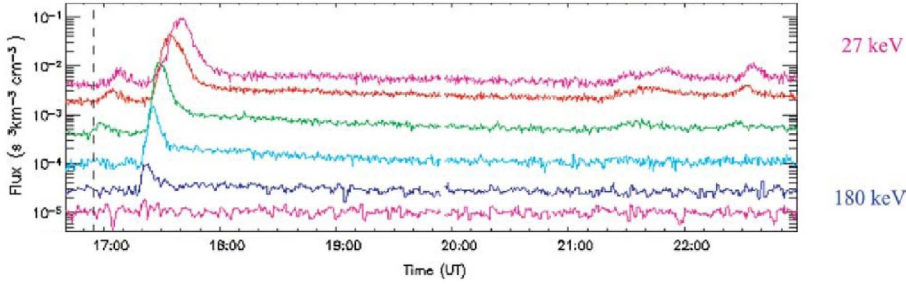


Fig. 2. Electron time profiles for the impulsive event of 7 August 1999 observed by WIND at ≈ 1 AU. Energies range between 27–300 keV (the blue label refers to the blue line at 180 keV). The dashed vertical line indicates the time at which the event started, the peak in the electron distribution is delayed of few minutes, each tickmark corresponding to ≈ 9 min (adjusted from Lin, 2005).

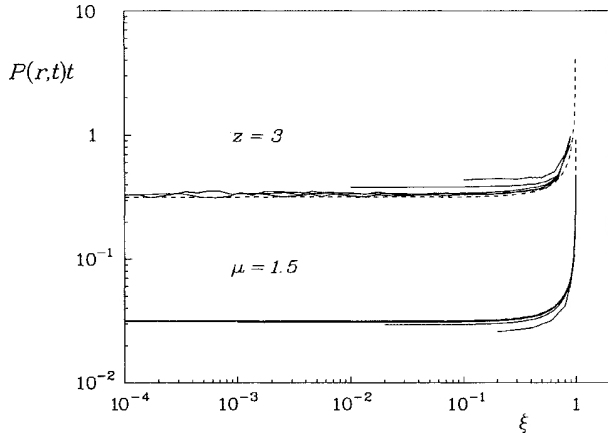


Fig. 3. Propagator for the ballistic regime as a function of the dimensionless variable $\xi = r/vt$ (adapted from Zumofen and Klafter, 1993). Note that here the time is increasing from the right to the left. The dashed line represents the analytical expression in Eq. (3), solid lines are results of numerical simulations.

Lee, 1980; Lee and Fisk, 1982) and the propagator is Gaussian. If we assume a superdiffusive transport in the heliosphere and we put the expression in Eq. (2) into Eq. (4), the particle time profile becomes

$$f(0, E, t) \propto \frac{1}{(-t)^{\mu-2}} = \frac{1}{(-t)^{\gamma}}, \quad (5)$$

having assumed that the spacecraft is at $x=0$ and that the shock starts at $t_0 = -\infty$, so that $t < 0$ (Perri and Zimbardo, 2007, 2008). Analysing data of electron fluxes detected by Ulysses spacecraft at 5 AU in the period between the beginning of 1992 (latitude 13° S) and the late 1993 (latitude 41° S), when the solar wind conditions were rather steady, we find that in the tails of electron fluxes time profiles are well described by a power law instead of an exponential decay. As an example, in Fig. 1 we report in log–log scale the electron profiles for the CIR shock of 22 January 1993. Values of the exponent of the power law γ go in the range 0.81–0.98, giving rise to $\mu = 2.81 - 2.98$ and $\langle r^2(t) \rangle \simeq t^{1.02} - t^{1.19}$, which indicates a superdiffusive regime. Assuming that the analysed data are counting, we adopt a Poissonian statistics and

we set errors for the y-axis values to \sqrt{y} . The chi-square test has also been applied, showing that the power law fits better the data with respect to the simple exponential one (Perri and Zimbardo, 2008). In Fig. 1 we can see a break in the power-law behaviour at 10 hrs from the shock front. We argue that this break is related to the different form that the propagator assumes for small r . Other examples of electron profiles corresponding to superdiffusive transport are given in Perri and Zimbardo (2008).

The WIND 3-D P instrument detected at nearly 1 AU high energy electrons (from few keV to 300 keV) coming from impulsive events from the Sun as solar flares or coronal mass ejection driven shocks (Lin, 2005). This kind of events have been accurately studied in order to obtain informations on the injection source near the Sun. These observations highlighted a scatter-free motion of electrons up to 1 AU. In Fig. 2 we show the impulsive event of 7 August 1999 as it has been reported by Lin (2005); it is characterized by rapid and symmetric rise and decay, which indicate a free motion, indeed scattering would tend to create a slowly decaying tail. Nearly scatter-free motion can also be inferred by comparing the velocity of particles with the time delay between electron injection in the corona, indicated by the vertical dashed line and deduced from the start time of type III radio bursts (Lin, 2005), and the onset time at WIND. On the other hand, the observed time profiles reported in Fig. 2 compare well with propagator for ballistic transport, given by Eq. (3) and reported in Fig. 3. In Fig. 3, the propagator is plotted as a function of the scaling variable $\xi = r/vt$; when $\xi = 1$, i.e. $r = vt$, the propagator diverges and a spike is found in the plot (since $\xi = r/vt$, for a given position r , in Fig. 3 time increases in the opposite direction with respect to Fig. 2). Especially for the green and the light-blue curves, electron time profiles seem to recover the scatter-free propagator. For ballistic transport the jump probability exponent μ satisfies $1 < \mu < 2$, then we can suppose that this exponent influences the width of the spike in Fig. 3, i.e. the larger μ , the larger the spike. This also emphasizes that there is not a sole way to obtain ballistic transport: some particles can be scattered back, giving rise to a small tail of late particles (see Fig. 2), in agreement with the focused transport theory (Ruffolo, 1995), although the majority of particles moves fast enough to have $\langle r^2(t) \rangle \propto t^2$.

4 Conclusions

We investigated the possibility of having an anomalous transport for particles accelerated in the interplanetary medium and near the Sun. We considered the relevant non Gaussian propagators that describe the evolution in space and time of particle fluxes after being emitted by the source. Looking at electron time profiles related to SEP events, Lin (2005) showed that electron propagation in the solar wind is ballistic; we can confirm this result by comparing WIND observations with the propagator for a ballistic transport, which is also characterized by a tail of weakly scattered particles. Also, we found a superdiffusive motion of energetic electrons when studying the electron fluxes at some distance from a CIR shock; latter fluxes are power laws in the tail with slope $\gamma = \mu - 2$, rather than simple exponential decays as expected in the case of normal diffusion. On the other hand, Perri and Zimbardo (2008) also studied the time profiles of protons accelerated at CIR shocks, finding a diffusive regime. They argued that these differences between electrons and protons can be due to the Larmor radii values. Indeed, they found that the magnetic field variations sampled by electrons are much weaker than those experienced by protons, leading to a sensitive decreasing of pitch angle diffusion.

Propagation of particles in the interplanetary medium is a very complicated subject because all parameters change with the heliocentric distance and with time. A complete model should take into account the spatial and temporal variation of the source, the convection and the adiabatic deceleration, the variation of the scattering mean free path λ_{\parallel} with radial distance and the particle rigidity (Ruffolo, 1995; Kallenrode, 2001; Lintunen and Vainio, 2004; Lee, 2005). Our findings stress the importance of considering the possibility of non diffusive regimes when studying the propagation of cosmic rays in space. From this point of view it is necessary to take into account that, at least for electrons at 40–300 keV, the standard DSA theory needs to be extended to the cases of superdiffusive transport.

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