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An explanation of the Pioneer anomaly involving accelerated atomic clocks

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Abstract. The Pioneer anomaly stands for unexplained frequency shifts of the Doppler radio-tracking signals received at the ground stations from the Pioneer 10 and 11 spacecraft in disagreement with expectations based on model calculations. We consider here observations of Pioneer 10 at heliocentric distances between 40 ua and 70.5 ua over a time interval of 11.55 years from 1987 to 1998. The anomaly has been interpreted in the literature either as a Doppler shift caused by an apparent spacecraft deceleration not accounted for by known effects, or as an unexpected clock acceleration of the frequency standards at the ground stations. The reasons for the anomalous behaviour are not understood in both cases. Based on a gravitational impact model – requiring a secular mass increase of all massive bodies – a solution is proposed that implies a clock acceleration with a value close to that of the Hubble constant.

1 Introduction

The large number of references in a recent presentation of the Pioneer Anomaly in Living Reviews in Relativity (cf. Turyshev and Toth, 2010, and references therein) shows a great interest of the scientific community in this phenomenon. It has been detected as an apparent trajectory anomaly of the Pioneer 10 and 11 spacecraft (cf. e.g. Anderson et al., 1998, 2002; Turyshev et al., 2006; Turyshev and Toth, 2009). The observations of the anomalous frequency shifts could be interpreted as a real – but unexplained – deceleration of the approximately anti-sunward directed heliocentric spacecraft velocity, $\boldsymbol{v}_\mathrm{p}$. A (nearly) sunward-directed force (in addition to all other *known effects*[1]) could cause this deceleration at a level of

$$a_\mathrm{p} = -(8.74 \pm 1.33) \times 10^{-10}\ \mathrm{m\ s^{-2}}\ . \qquad (1)$$

Alternatively, the drift rate of the frequency shift would be compatible with a clock acceleration at the ground stations – also unaccounted – of

$$a_t = (2.92 \pm 0.44) \times 10^{-18}\ \mathrm{s^{-1}}\ , \qquad (2)$$

although a true trajectory anomaly together with an unknown systematic spacecraft effect was considered to be the most likely interpretation (Anderson et al., 2002). Turyshev and Toth (2010) have concluded that the nature of the anomaly remains unexplained, even though quite a few potential reasons ranging from spacecraft effects to new physics have been proposed since 1998, most of them are documented in their extensive reference list. However, articles by Petry (2005); Fahr and Siewert (2007, 2008) and Hajdukovic (2010), for instance, have not been included therein. Petry deduces from his analysis of the Pioneer anomaly in the framework of a flat space-time geometry that an acceleration has to be expected opposite to the direction of the velocity. On the other hand, Fahr and Siewert discuss the concept of the Einstein–Straus vacuole and the dynamics of the local space-time metric with the creation of local mass. Hajdukovic assumes gravitational repulsion between matter and antimatter as well as virtual particle-antiparticle pairs that cause the Pioneer anomaly via their polarization in the gravitational field of the Sun.

2 Observations

Anomalous frequency shifts of the Doppler radio-tracking signals were observed for both Pioneer spacecraft and have

[1] The gravitational pull of the total mass of the inner solar system of 1.992×10^{30} kg reduced the speed of the Pioneer 10 spacecraft on its hyperbolic escape trajectory from $v_\mathrm{p}(r_0) \approx 13\ \mathrm{km\,s^{-1}}$ at $r_0 = 40$ ua to $v_\mathrm{p}(r_1) \approx 12\ \mathrm{km\,s^{-1}}$ at $r_1 = 70.5$ ua.

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been discussed in great detail by Anderson et al. (2002) and Turyshev et al. (2006) among others. In this contribution, only Pioneer 10 (launched on 2 March 1972) will be considered during the time interval, (t_0, t_1), between 3 January 1987 and 22 July 1998 ($t_1 - t_0 \approx 3.6 \times 10^8$ s), while the spacecraft was at heliocentric distances between $r_0 = 40$ ua and $r_1 = 70.5$ ua. The unaccounted frequency shift drifted with a nearly constant rate[2] towards higher frequencies (i.e. to the *blue* side of the spectrum and thus has a positive sign). The frequency drift rate was obtained as a result of coherent Doppler observations and had been deduced from data of the Deep Space Network (DSN) S-band communication system near 2 GHz using the DSN sign convention (Anderson et al., 2002, see Note 38 therein). The analysis carried out by the Pioneer team took into account the large red shift caused by $\boldsymbol{v}_{\rm p}$ and all other known contributions in calculating a model frequency, $\nu_{\rm model}(t)$. It was based on a constant clock frequency[3] f_0 and the elapsed time Δt since the initial epoch[4] $t = t_0$. In general, Δt is much larger than the signal round-trip time $\delta t = t_{\rm r} - t_{\rm s}$, where $t_{\rm r}$ and $t_{\rm s}$ are the signal reception and transmission times, respectively. Repeated observations and calculations at times $t = t_0 + \Delta t$ indicated a nearly uniform increase of the observed frequency with respect to the expected one of

$$\nu_{\rm obs}(t) - \nu_{\rm model}(t) = 2\,\dot{f}\,\Delta t \quad (3)$$

with $\dot{f} = 5.99 \times 10^{-9}$ Hz s^{-1} (cf. Turyshev et al., 2006). The values in Eqs. (1) and (2) as well as $\dot{f}$ in Eq. (3) have been deduced by the Pioneer team and will be accepted as valid without further discussion at this stage; supplemented by the assumption that a *prosaic* explanation, such as an additional spacecraft effect balancing the anomaly – as suggested, for instance, by Katz (1999); Murphy (1999); Scheffer (2003) – would not have passed unnoticed. As long as the Pioneer team has not formally declared that the anisotropic radiation of the radioisotope thermoelectric generators is quantitatively causing the unmodelled frequency drift of both spacecraft, the search for a solution appears to be pertinent in view of the fact that "Only scarce documentation is available about the exact geometry of the Pioneer 10 and 11 spacecraft" is stated in the most recent review on "The Pioneer Anomaly" (Turyshev and Toth, 2010). Turyshev et al. (2011) now report indications of a temporal variation of the anomaly[5], which support anisotropic heat emission from the spacecraft as a possible cause of the anomalous deceleration. The authors leave the question open "... whether or not a statistically significant anomalous acceleration still remains ...". Under these conditions, we tend to believe that a concluding statement "... unless new data arises, the puzzle of the anomalous acceleration of the Pioneer probes can finally be put to rest" by Francisco et al. (2011) might be somewhat premature. Nevertheless, we take into account the indications that some or all of the anomalous effect might have been caused by more anisotropic emission of the total nuclear power (2.2 kW in 1987 decaying to 1.9 kW in 1998) than had been assumed in earlier studies (cf. Bertolami et al., 2008; Rievers et al., 2009; Turyshev and Toth, 2009; Rievers and Lämmerzahl, 2011), and will consider the potential consequences in Sect. 4.

In addition, it should be kept in mind that other anomalies have been observed in the solar system, although under less favourable conditions compared to those of the Pioneer probes (Anderson et al., 2008; Lämmerzahl et al., 2008).

3 Interpretations

The temporal derivation $\dot{f}$ of $f(t)$ can be considered to be the drift rate of a receiver-clock reading at time t with respect to a constant transmitter-clock frequency f_0 obtained in a one-way Doppler observation[6] (Toth, personal communication; see also Sect. 3.1). A definition, appropriate for the present discussion, is

$$f(t) = f_0 + \dot{f}\,\Delta t\ , \quad (4)$$

such that $\nu_0 = f_0$ is the emitted frequency and $\nu(t) = f(t)$ would be the received one at the spacecraft.[7]

The basic Eq. (3) has been re-written below in different formats depending upon the choice of the interpretation. The corresponding discussions can considerably be simplified, if all *known* frequency variations are no longer included in the equations. This is possible, because they have already been taken care of by the model calculations of the Pioneer team. The observed and calculated frequencies without known contributions to their variations will be denoted by $\nu_{\rm anom}$ and $\nu_{\rm mod}$ in this convention. This is equivalent to setting all square-bracket terms to zero in the modified Eqs. (5 a) and (6 a).

I. A Doppler shift interpretation is best described by the following two equations:

$$[\nu_{\rm obs}(t) - (f_0 + 2\,\dot{f}\,\Delta t)] - [\nu_{\rm model}(t) - f_0] = 0 \quad (5)$$

[2] Small periodic variations, recently discussed in detail by Levy et al. (2009), will not be considered here.

[3] A frequency $f(t_0) = f_0 \approx 2.05$ GHz is consistent with the values of a_p, $\dot{f}$ and $a_t = \dot{f}/f_0$ in Eqs. (1), (3) and (2). The frequency translation ratio of 240/221 between up-link and down-link communications is only of technical interest.

[4] Unless explicitly indicated, the time definition of the Système International d'Unités (SI) based on a hyperfine transition in ^{133}Cs will be used – in some cases as an approximation.

[5] Acceleration estimates between $\approx (-8.3$ and $-7.2) \times 10^{-10}$ m s^{-2} are given for the time interval considered here.

[6] The actual measurements were performed in a two-way Doppler mode according to Eq. (3) assuming equal effects during the up-link and down-link propagation times.

[7] For reasons of clarity, clock frequencies will be denoted by the symbol f and radio signal frequencies by ν, both measured in units of hertz.

$$[\nu_{\rm anom}(t) - (f_0 + 2\,\dot{f}\,\Delta t)] - [\nu_{\rm mod}(t) - f_0] = 0 \qquad (5\,a)$$

Equation (5 a) will be treated in Sect. 3.1.

II. An alternative formulation lends itself to an interpretation involving a general acceleration of both the transmitter and the receiver clocks on the ground. It will be discussed in Sect. 3.2:

$$[\nu_{\rm obs}(t) - (f_0 + \dot{f}\,\Delta t)] - [\nu_{\rm model}(t) - (f_0 - \dot{f}\,\Delta t)] = 0 \qquad (6)$$

$$[\nu_{\rm anom}(t) - (f_0 + \dot{f}\,\Delta t)] - [\nu_{\rm mod}(t) - (f_0 - \dot{f}\,\Delta t)] = 0 \qquad (6\,a)$$

3.1 Doppler effect

As mentioned above, the model calculations for the first case assumed a constant reference frequency f_0. This implies $\nu_{\rm mod}(t) = f_0$ and, of course, a constant transmitter frequency $\nu_0 = f_0$. From Eq. (5 a) it follows that $\nu_{\rm anom}(t) = \nu_0 + 2\,\dot{f}\,\Delta t$, which can conveniently be interpreted as a two-way Doppler shift caused by a reflection from a spacecraft moving towards the ground station(s) with an anomalous speed of

$$v_{\rm anom}(t) = a_{\rm p}\,\Delta t\ , \qquad (7)$$

where the numerical value of $a_{\rm p}$ is given in Eq. (1). It should be re-iterated that Δt is different from the signal round-trip time δt. The relativistic Doppler effect (Einstein, 1905) yields an anomalous frequency behaviour of

$$\nu_{\rm anom}(t) = \nu_0\,\frac{1 - \beta_{\rm anom}(t)}{1 + \beta_{\rm anom}(t)}\ , \qquad (8)$$

where $\beta_{\rm anom}(t) = v_{\rm anom}(t)/c_0$ and c_0 is the speed of light in vacuum. In view of the slow speeds involved, the frequency shift can be approximated by the non-relativistic Doppler equation

$$\nu_{\rm anom}(t) - \nu_0 \approx -2\,\nu_0\,\frac{a_{\rm p}}{c_0}\,\Delta t\ , \qquad (9)$$

and a comparison with Eq. (5 a) then shows that it is

$$\dot{f} = -\nu_0\,\frac{a_{\rm p}}{c_0} = -f_0\,\frac{a_{\rm p}}{c_0}\ , \qquad (10)$$

and differentiation of Eq. (9) yields

$$\dot{\nu}_{\rm anom} \approx 2\,\dot{f}\ . \qquad (11)$$

It can, therefore, be concluded that the introduction of a deceleration, $a_{\rm p}$, of Pioneer 10, in addition to all other known effects, removes the term representing the anomaly. However, it is not the only interpretation as has already been pointed out by Anderson et al. (1998).

3.2 Clock acceleration

An interpretation of the anomaly with the help of the clock acceleration a_t in Eq. (2) and *without anomalous spacecraft motions* can be obtained by considering Eq. (6 a). Together with Eq. (4), written in the form

$$f(t) = f_0\left(1 + \frac{\dot{f}}{f_0}\,\Delta t\right) = f_0\,(1 + a_t\,\Delta t)\ , \qquad (12)$$

we draw conclusions as described in the following subsections.

3.2.1 Long-term considerations

Eq. (6 a) describes the situation, where a frequency $\nu_{\rm anom} = f_0 + \dot{f}\,\Delta t$ will be obtained with a variable model frequency $\nu_{\rm mod}(t) = f_0 - \dot{f}\,\Delta t$. It is qualitatively not different from Eq. (5 a), if the reference frequency is again f_0. Assuming, however, a clock acceleration, a_t, requires to refer the signals to the frequency $f(t)$ of Eq. (12) as reference.[8] Consequently, Eq. (6 a) has to be expressed as

$$\{\nu_{\rm anom}(t) - f(t)\} - \{\nu_{\rm mod}(t) - [f(t) - 2 f_0\,a_t\,\Delta t]\} = 0, \qquad (13)$$

in which both curly bracket terms must be zero. The second term is zero with a model frequency of

$$\nu^*_{\rm mod}(t) = \nu_{\rm mod}(t) + 2\,f_0\,a_t\,\Delta t = f(t)\ , \qquad (14)$$

i.e. the clock frequency at time t determined by Eq. (12). The constraint of Eq. (13) is then met with $\nu_{\rm anom}(t) = \nu^*_{\rm mod}(t)$. The clock acceleration, therefore, offers a way to understand the frequency anomaly without anomalous spacecraft behaviour.[9] Such an option is not excluded, because Doppler range-rate measurements cannot differentiate between a spacecraft deceleration and a clock acceleration (cf. Turyshev and Toth, 2010); and telemetry range data are not available for the Pioneer spacecraft (Anderson et al., 2002).

3.2.2 Round-trip signals

The frequency and clock drifts can also be applied to round-trip signals from a ground station to the spacecraft and back to Earth (although many years of observations went into the determination of $\dot{f}$ as outlined in Sect. 3.1). Note that – according to the assumption in Sect. 3.2 – no anomalous spacecraft motion will now be included in the model calculations. After the signal has been sent off at $t_{\rm s}$, nothing unexpected is assumed to happen to it[10] and, consequently, it is $\nu_{\rm anom}(t_{\rm r}) = f(t_{\rm s})$, where $\nu_{\rm anom}(t_{\rm r})$ is the observed and

[8] In line with the approximation in Sect. 2, it is $t \approx t_{\rm s} \approx t_{\rm r}$, i.e. the signal round-trip delay will be neglected.

[9] Any clock acceleration will – to a certain extent – also affect the apparent motion of the spacecraft. An estimate will be derived in Appendix B, which shows that the effect can be neglected.

[10] See Appendix B for a justification.

adjusted frequency at reception after a total transit time δt. Since the clock frequency increased by $a_t\,\delta t$ according to Eq. (12), a relative red shift of the signal frequency would be expected. This is not in conflict with the statements in Sects. 3.1 and 3.2.1 as long as the condition $\Delta t \gg \delta t$ is fulfilled. Observations with $\Delta t \approx \delta t$ are, however, not available with the required accuracy and thus cannot be used to distinguish between a clock-acceleration hypothesis or a true anomalous spacecraft motion, which would result in a blue shift.

4 Proposed solution

The Pioneer anomaly, together with observed fly-by anomalies (for a recent discussion see Anderson et al., 2008), and some other – possibly related – unexplained phenomena, led Lämmerzahl et al. (2008) to raise the question, whether the physics within the solar system is really understood. Considering the conclusions in most of the recent articles on this topic, the answer to the question appears to be *No*. It is, therefore, only appropriate to present a solution for the Pioneer anomaly that is based on a proposed gravitational model involving the exchange of massless entities – called quadrupoles – between elementary particles. The model has been designed to emulate Newton's law of gravity under the assumption that there are no far-reaching fields, and the exchange of energy and momentum has to be accomplished by those massless entities – in analogy with the transfer of energy and momentum by photons. In order to model the attraction, a re-emission of the absorbed quadrupoles with reduced energy and momentum was required. Consequently, the model predicts the most significant aspect in our context, i.e. a secular mass increase of all massive particles in the Universe governed by the law of gravitation (fuelled by a background quadrupole flux) according to the equation

$$M(t) = M_0\,\exp[A\,(t-t_0)]\,, \qquad (15)$$

where M_0 is the (rest) mass of a body at time t_0 and M at some later time t. It should be noted that the model has some features in common with the impact theory of Nicolas Fatio de Duillier presented to the Royal Society of London in 1690 (see, e.g. Bopp, 1929; Gagnebin, 1949). The gravitational attraction in our modified impact model is, however, not a consequence of Fatio's shadow effect, but of the energy absorption and mass creation.

Mass creation hypotheses albeit operating under different processes have been discussed by many authors. Hoyle (1948) quotes Jeans (1928) with a remark that galaxies "... appear as points at which matter is being continually created", before suggesting a stationary Universe, in which "Neutron creation appears to be the most likely possibility." Creation of neutrinos is mentioned by Massa (1994) and ejection of new matter from nuclei of galaxies by Arp et al. (1990). An increase of elementary particles in the Universe with time had also been considered by Dirac (1937). These topics are reviewed by Fahr (1995) – mentioning, in particular, that an electron mass increase might result from Mach's principle of inertia and the corresponding variation of the so-called Rydberg constant could lead to the cosmological red shift. Fahr and Heyl (2007) suggest that a decay of the vacuum energy density creates mass in an expanding Universe.

Our estimates for the quantity A in Eq. (15) were based on the requirement to emulate Newton's law of gravity. They ranged from $A \approx 6\times10^{-29}\,\mathrm{s}^{-1}$ up to $A \approx H_0 \approx 2.4\times 10^{-18}\,\mathrm{s}^{-1}$, the Hubble constant. No mensurable effect over the age of a Big-Bang Universe should be detectable with present-day uncertainty margins at the lower limit, whereas near the upper limit, an accelerated expansion of the Universe (cf. Perlmutter et al., 1999; Ries et al., 2001) would not be required to explain the type Ia supernovae observations. In any case, the quantity A is so small that Eq. (15) can be written in very good approximation as the first two terms of the exponential function expansion

$$M(t) \approx M_0\,[1 + A\,(t-t_0)] \qquad (16)$$

for the time intervals under consideration.

The Pioneer anomaly appears to offer a good opportunity for a test of any gravitational model, because the extreme accuracies achievable in frequency and time measurements are combined with very long temporal and spatial baselines. The quadrupole model outlined above will, therefore, be applied to the solar system, although one important aspect, namely, the transition from elementary particles to large conglomerations of mass, has not yet been unambiguously achieved in the model. The assumption here will be that the scaling law is just proportional to the mass.

Atomic frequencies are linearly related to the Rydberg constant

$$R_\infty = 10\,973\,731.568\,527\ \mathrm{m}^{-1} \quad (u_\mathrm{r} = 6.6\times10^{-12}) \qquad (17)$$

(Codata recommended value 2006). The impact model, however, implies that R_∞ increases together with the electron mass[11] as a function of time

$$R_\infty = \frac{\alpha^2\,c_0}{2h}\,m_\mathrm{e}\,[1 + A\,(t-t_0)]\,, \qquad (18)$$

where α is the fine structure constant, h the Planck constant and m_e the electron mass at t_0. According to the Rydberg–

[11]The other quantities in Eq. (18) are assumed to be constant. Constraints on the variation of α with time are orders of magnitude below that of A (cf. Uzan 2003; Lea 2008). The distinction between dimensional constants, such as the Rydberg constant, and dimensionless ones has been emphasized by Karshenboim and Peik (2008).

Ritz formula with a proton mass m_p at t_0, the optical transition frequencies are

$$f(n_2,n_1) = c_0\, R_\infty \left(1+\frac{m_\mathrm{e}}{m_\mathrm{p}}\right)\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right); \quad n_2 > n_1 \qquad (19)$$

for hydrogen[12]. The Fermi energy and its corrections (relevant for the hyperfine-structure transition used in hydrogen masers) are proportional to the Rydberg constant as well (cf. Fermi, 1930; Goudsmit, 1931; Nafe et al., 1947; Karshenboim and Ivanov, 2002). One would consequently expect an increase of the frequency of the clocks with time at the DSN ground stations. From Eq. (18), it thus follows that

$$f(t) = f(t_0)\,[1 + A\,(t-t_0)] \qquad (20)$$

and a comparison with Eq. (12) indicates that the Pioneer anomaly as defined by Eqs. (1) and (2) can be understood by equating the clock acceleration a_t with A and H_0 – within the uncertainty margins of H_0 (cf. e.g. Freedman and Madore, 2010) and of a_t – while assuming a nominal spacecraft motion along the trajectory. In this context, it is interesting to note that Iorio (2007) concluded that it seems to be "difficult to realistically consider the possibility that some modification of the current laws of gravity may be the cause of the Pioneer anomaly ..." .

Whereas the choice $a_t \approx H_0$ is near the upper limit of the estimates of the quantity A governing the secular mass accretion and could not be increased within standard concepts of the Universe, smaller values of a_t which might eventually result from a revised analysis of the observations (cf. Sect. 2) can be accommodated without difficulty in view of the wide range of A. In this sense, the correspondence of H_0 with A in Eqs. (15) and (16) would only be directly relevant if Eq. (3) could be quantitatively confirmed.

It is to be noted that the clock acceleration is not related to the spacecraft and, therefore, the anomaly would not be subject to any influence of the changing distance of Pioneer 10 from the inner solar system – in accordance with earlier observations (see Anderson et al., 2008, 2002; Turyshev and Toth, 2010), but, as mentioned before, a relative variation of ≈ 15 % has been reported very recently (Turyshev et al., 2011).

The secular mass increase modifies the gravitational potentials in the solar system. The resulting effects on the Pioneer spacecraft are, however, many orders of magnitude smaller than the observed anomaly, and would amount to an additional deceleration of $\approx -2\times10^{-15}\,\mathrm{m\,s^{-2}}$ at the end of the selected time interval. The influence on the planetary system could be more significant, although Fahr and Siewert (2007) believe that there is no observable conflict with conventional celestial mechanics for $\dot{M}/M = H_0$. This topic is, however, very involved and has been discussed for many decades. An increase of the mass of the central body will lead to a decrease of the mean orbital radius of a revolving body with constant mass (cf. Strömgren, 1903; See, 1911). Kepler's third law then stipulates that an increase in Newton's "constant" of gravity, G_N, will have the same effect provided the masses of both bodies do not change. If this condition is not fulfilled, such a conclusion cannot be drawn without detailed considerations. This applies, in particular, to recent results published in relation to a possible temporal variation of G_N: Lunar-laser-ranging data yield values of $\dot{G_\mathrm{N}}/G_\mathrm{N}$ from $(-0.2\pm1.2)\times10^{-20}\,\mathrm{s^{-1}}$ to $(1.9\pm2.2)\times10^{-20}\,\mathrm{s^{-1}}$ (Merkowitz, 2010; Hofmann et al., 2011); white-dwarf and neutron-star observations give $-5.7\times10^{-20}\,\mathrm{s^{-1}}$ (García-Berro et al., 2011) and $|\dot{G_\mathrm{N}}/G_\mathrm{N}| < 1.9\times10^{-19}\,\mathrm{s^{-1}}$ (Reisenegger et al., 2001), respectively.

We performed a preliminary estimate in response to reports of an increase of the mean Sun-Earth distance (see, for instance, Krasinsky and Brumberg, 2004), which indicated that A should be significantly smaller, but still near the upper limit of the range from $6\times10^{-29}\,\mathrm{s^{-1}}$ to $2.4\times10^{-18}\,\mathrm{s^{-1}}$ given above. If this could be confirmed, the Pioneer anomaly would have to be smaller than specified in Eq. (3).

5 Discussion and conclusions

The Doppler data are obtained in relation to atomic clocks. According to Eq. (20), their frequencies increase with time in the clock-acceleration scenario and thus there is less time for the spacecraft to be slowed down by the gravitational attraction of the inner solar system than assumed in the model calculation based on a constant f_0. The difference in speed is, however, so small that the resulting red shift can be neglected as shown in Appendix B.

A single round-trip measurement, if feasible, would also show a red shift of the anomalous signal compared to the increased atomic clock frequency at the receiving station, but the contribution is proportional to $\delta t/\Delta t$ and thus very small (see again Appendix B).

So, what produces the frequency increase of $2\,\dot{f}\,(t_1 - t_0) = 4.34\,\mathrm{Hz}$ over the observational period discussed under the assumption of an atomic clock acceleration? According to Eq. (6 a), it is the inappropriate choice of the reference frequency f_0 under these circumstances. This reference is too small by $\dot{f}\,\Delta t = f_0\,a_t\,\Delta t$ both at the transmitting and receiving station(s), giving an apparent blue shift of the signal of twice that amount, i.e. $2\times2.17\,\mathrm{Hz}$ after 11.55 years. This is remedied in Eqs. (13) and (14) with a reference frequency $f(t)$.

A gravitational impact model together with a growth time of the secular mass increase of the order of the inverse Hubble constant and a corresponding clock acceleration can provide an explanation for the Pioneer anomaly as presently characterized by Eq. (2). Under the same assumption, Fahr

[12] The ratio $\mu = m_\mathrm{e}/m_\mathrm{p}$ would not be directly affected by the secular mass increase. The fine-structure transitions are also proportional to R_∞.

and Siewert (2007) also find a mass creation rate from their local space-time metric, which is in accordance with Eq. (16). It remains to study (in the framework of this model) the other unexplained observations listed by Lämmerzahl et al. (2008) in the hope that they might indeed not be isolated phenomena. It is clear that $a_t \approx A \approx H_0$, if confirmed, would have important ramifications for many cosmological questions. However, the impact model could accommodate a wide range in A, and thus the association with H_0 is only relevant as long as Eq. (2) is valid.

Appendix A

Atom and photon clocks

Among other authors, Rañada and Tiemblo (2008) emphasized the need to distinguish between different clocks. They defined astronomical and atomic clocks pointing out that the Pioneer anomaly might provide a means to determine their actual relation.

Such a concept is modified in this study: The atomic frequency of a maser clock is assumed to increase according to Eq. (12). A photon frequency, on the other hand, will be considered to be constant in an inertial system, because there cannot be a rest mass increase of a photon. A photon clock[13] can be defined by counting electromagnetic wave periods. In a *Gedankenexperiment* a signal will be emitted at t_0 with ν_0 and compared with atomic clocks after successive reflections (and corrections for known effects) between the spacecraft and the ground station.

If the advancement of time is given by the number N of cycles (counting from $t = t_0$) multiplied by their period $T = 1/\nu$, a photon time with constant $\nu = \nu_0$ is easy to formulate as

$$t^{\mathrm{photon}}(N) = t_0 + \frac{N}{\nu_0} = t_0 + \Delta t \; . \tag{A1}$$

For the atomic time, the variation of the frequency f with N has to be taken into account. This is done in the following equations by using the photon time as baseline for the calculation of time differences, thus neglecting higher order terms:

$$\frac{\mathrm{d}t^{\mathrm{atom}}(N)}{\mathrm{d}N} = \frac{1}{f(N)} \approx \frac{1}{f_0 + \dot{f}\,\Delta t}$$
$$\approx \frac{1}{f_0}\left(1 - a_t\,\frac{N}{f_0}\right) = \frac{1}{f_0} - a_t\,\frac{N}{f_0^2} \; . \tag{A2}$$

Integration over N gives together with Eq. (A1) and an integration constant t_0 for $N = 0$, when both clocks are thought to be synchronized, i.e. $\nu_0 = f_0$

$$t^{\mathrm{atom}}(N) \approx t_0 + \Delta t - \frac{a_t}{2}\,\frac{N^2}{\nu_0^2}$$
$$= t^{\mathrm{photon}}(N) - \frac{a_t}{2}\,(\Delta t)^2 \; . \tag{A3}$$

This is equivalent to Eq. (64) of Anderson et al. (2002) and leads to $t^{\mathrm{photon}} - t^{\mathrm{atom}} = 0.19$ s after 11.55 years.

[13]Not to be confused with an optical clock or a photon-reflection clock, Einstein's *Lichtuhr*.

Appendix B

Clock acceleration and spacecraft trajectory

The spacecraft speed as a function of N in the photon time system is given by

$$v_{\mathrm{p}}(N) = \frac{\Delta r}{\Delta t} = \nu_0\,\frac{\Delta r}{\Delta N} = \nu_0\,\frac{r(N+\Delta N) - r(N)}{\Delta N} \; . \tag{B1}$$

It is thought to be the true heliocentric velocity. Taking into account the values in footnote 1, the spacecraft speed is $v_{\mathrm{p}} \approx 12\,\mathrm{km\,s^{-1}}$ and is approximately constant for the purpose of determining the differential speed with respect to the atomic time system. The apparent difference can then be obtained with the help of Eq. (A3) as

$$v_{\mathrm{p}}^{\mathrm{atom}}(N) - v_{\mathrm{p}} \approx v_{\mathrm{p}}\,a_t\,\Delta t \; . \tag{B2}$$

The true velocity v_{p} is thus slower than $v_{\mathrm{p}}^{\mathrm{atom}}$ leading to a small apparent blue shift of 1.7×10^{-4} Hz at the end the observational period, which is only a fraction of 4×10^{-5} of the anomalous Pioneer shift. The concern raised in footnote 9 thus is, indeed, not critical. It might be of interest to consider as well the difference in the location of the spacecraft at that time. It is only 2300 m, whereas a true trajectory anomaly would displace the spacecraft by about 57 Mm. Another apparent frequency shift stems from the changing atomic clock frequency while the signal is on a round trip to the spacecraft. A numerical example for a Pioneer distance of 70.5 ua, corresponding to a round-trip time of $\delta t \approx 7 \times 10^4$ s, would result in a red shift of $\delta\nu = -\dot{f}\,\delta t \approx -4 \times 10^{-4}$ Hz.

Note, however, that in the *Gedankenexperiment* of Appendix A the signal emitted at t_0 would appear to be anomalous shifted by $\Delta\nu = -2.17$ Hz, i.e. towards red, for $\Delta t = 3.64 \times 10^8$ s at the end of the observational interval.

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