

Three dimensional model of the interplanetary magnetic field and 27-day variation of the galactic cosmic ray intensity

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Abstract. We present a model of the 27-day variation of the galactic cosmic ray (GCR) intensity for three dimensional (3-D) heliosphere with the heliolongitudinal and heliolatitudinal dependent radial solar wind speed for the period of 1995 - minimum epoch of solar activity ($A > 0$). In the present model we implement heliolongitudinal asymmetry of solar wind velocity reproducing as the sum of first and second harmonics depending on the heliolatitudes and the regular part of the solar wind velocity (V_0) changing versus heliolatitudes in accord to in situ measurements of the Ulysses spacecraft. We show that the range of changes of the sum of the first and second harmonics of the 27-day variation of the GCR intensity for Kiel neutron monitor is little less than expected from the modelling, however, they are comparable.

1 Introduction

Recently, we demonstrated that to model the 27-day variation of the galactic cosmic ray (GCR) intensity, there should be taken into account a consistent, divergence-free interplanetary magnetic field (IMF) derived from Maxwell's equations with the heliolongitudinally dependent solar wind velocity reproducing in situ observations (Alania et al., 2010). We believe that in situ measurements of the solar wind velocity and the IMF components only partly characterize electromagnetic properties of the whole vicinity of the interplanetary space where a formation of the 27-day variation of GCR intensity takes place. Due to complexity of the processes on the Sun and in the three dimensional (3-D) heliosphere one hardly could wait for high correlation between in situ observed components of the IMF and the expected divergence-free IMF components derived from Maxwell's

equations with the heliolongitudinally dependent solar wind velocity reproducing in situ observations. In spite, in situ measurements are an unique information which should be considered as a basic data in any type of modelling. This paper is a continuation of study presented in Alania et al. (2010) tending to construct a 3-D model of the 27-day variation of the GCR intensity being able in certain scope to explain a behavior of the GCR intensity during in arbitrary Bartels' rotation (during 27-days). In contrast to Alania et al. (2010) in a present model we implement heliolongitudinal asymmetry of solar wind velocity reproducing as the sum of its first and second harmonics depending on the heliolatitudes and the regular part of the solar wind velocity (V_0) changing versus heliolatitudes in accord to in situ measurements of the Ulysses spacecraft. As a case study we consider the 27-day variation of the GCR intensity, solar wind velocity and the IMF for the minimum epoch of solar activity of 12 January–28 December 1995 corresponding to the Bartels'rotations (BR) 2205-2217 (Fig. 1). Figure 1 shows that there are not a clear regular changes of different parameters related with the Sun's rotation for the whole period of 1995. However, there are recognizable the 27-day variation during some BR. To show relative competency of our approach to the construction of the 3-D model of the 27-day variation of the GCR intensity, we consider as an example average data of 13 BR (about one year, 1995).

2 Modeling of the 27-day variation of the GCR intensity

For modelling of the 27-day variation of the GCR intensity we use stationary Parker's transport equation (Parker, 1965). In the presented model we assume that the stationary 27-day variation of the GCR intensity is caused by the changes of the solar wind velocity. We assume heliolongitudinal asymmetry of the solar wind speed corresponding to the in situ



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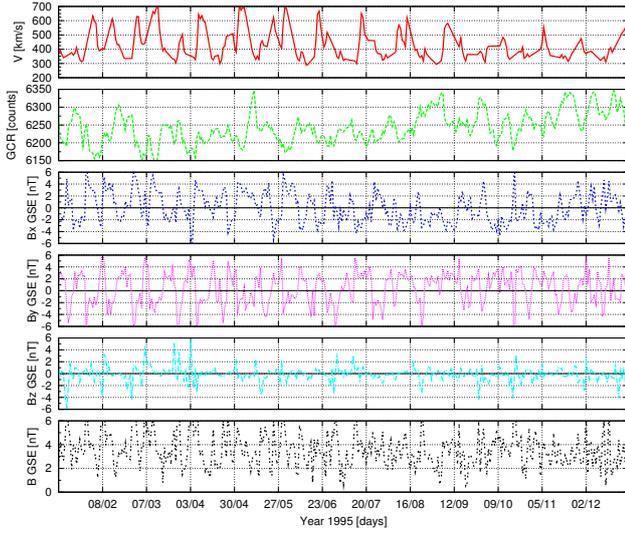


Fig. 1. Temporal changes of the daily solar wind velocity [OMNI, <http://omniweb.gsfc.nasa.gov/index.html>], GCR intensity from the Kiel neutron monitor and radial B_x , azimuthal B_y , latitudinal B_z components and magnitude B of the IMF [OMNI] for the minimum epoch of solar activity in the period of 12 January–28 December 1995 ($A > 0$) corresponding to the BR 2205-2217.

measurements at the Earth orbit (Fig. 2(a)) and versus heliolatitude as was shown by Ulysses for the minimum epoch of solar activity (McComas et al., 2000) (Fig. 2(b)). Presented in Fig. 2(a) are averaged by means of 13 BR, the daily data of the solar wind speed (points) and the approximation (dotted curve) of the first and second harmonic waves (27 and 14-day variations) during the period of 1995. An approximation of the solar wind speed included in the model has a form:

$$V_r = V_0 \cdot V_r(\varphi) \cdot V_r(\theta) \quad (1)$$

where the $V_r(\varphi) = 1 + 0.15 \sin(\varphi + 1.18) - 0.07 \sin(2\varphi + 0.64)$ is the approximation (dotted line in Fig. 2(a)) of the daily data of solar wind at the Earth orbit; $V_0 = 400$ km/s, $V_r(\theta)$ is extrapolation of solar wind velocity measured in the ecliptic to higher latitudes as showed by Ulysses for minimum conditions of solar activity (McComas et al. (2000)) approximated by formula (Fig. 2(b)):

$$V_r(\theta) = \begin{cases} -0.21\theta & 0^\circ \leq \theta \leq 40^\circ \\ 3.73 + 3.06 \text{ArcTg}(0.07 - \theta) & 40^\circ < \theta \leq 75^\circ \\ 1 & 75^\circ < \theta \leq 90^\circ \end{cases}.$$

We assume that the heliolongitudinal asymmetry of the solar wind velocity has maximum value at Earth's orbit and equals zero at Sun's poles regions, i.e. that the heliolongitudinal asymmetry changes versus heliolatitudes, as $\sin\theta$. Therefore, an expression included in the model of the 27-day variation of the GCR intensity has a form: $V_r(\varphi) \mapsto V_r(\theta, \varphi) = 1 + (0.15 \sin(\varphi + 1.18) - 0.07 \sin(2\varphi + 0.64)) \sin\theta$. Ascribing a decisive role in creation of the 27-day variation of the

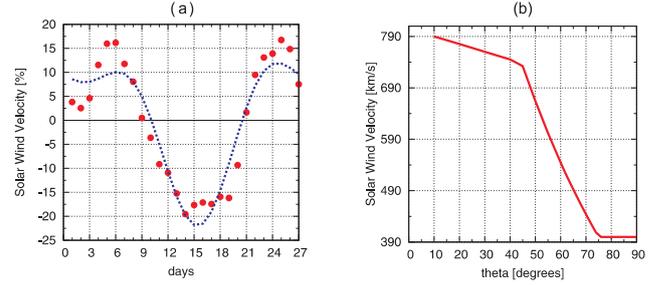


Fig. 2. (a) Temporal changes of the daily solar wind velocity at the Earth orbit (points) superimposed by means of 13 BR and approximation (dotted curve) by the sum of two harmonic waves for the period of 12 January – 28 December 1995 ($V_r(\varphi)$ in Eq. (1)); (b) Heliolatitude dependence of the radial solar wind speed for minimum epoch of solar activity in 1995 as showed by Ulysses for minimum conditions ($V_0 \cdot V_r(\theta)$ in Eq. (1))

GCR intensity to the heliolongitudinal asymmetry of the solar wind velocity we justify by the high correlation coefficient r ($r \approx -0.9$) between the changes of the sum of the first and the second harmonics of the 27-day variation of the GCR intensity (Fig. 4) and the solar wind velocity (Fig. 2(a)) for average BR during the period of 12 January–28 December 1995. At the same an ignorance of the role of the corotating interaction regions to some extent is justified by the negligible correlation coefficient r ($r \sim 0.1$) between the changes of solar wind velocity and magnitude of the IMF for considered period (Fig. 1). To solve Parker's transport equation (Parker, 1965) there is required the B_x , B_y and B_z components of the IMF corresponding to the changeable solar wind velocity. For this purpose Maxwell's equations should be solved for the solar wind velocity represented by Eq. (1). We consider Maxwell's equations:

$$\begin{cases} \frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{V} \times \vec{B}) \\ \text{div } \vec{B} = 0 \end{cases} \quad (2)$$

where B is the IMF strength, V -solar wind velocity, and t -time.

2.1 Numerical solution of Maxwell's equations

We assume that the changes of the solar wind velocity, the GCR intensity, B_x , B_y and B_z components of the IMF are quasi stationary, i.e. the distribution of the GCR density is determined by the time independent parameters. Therefore, we accept that in Eq. (2) $\frac{\partial B}{\partial t} = 0$. Also, we accept that average value of the heliolatitude component of the solar wind velocity V_θ equals zero. These assumptions lead to a system of scalar equations for the IMF's and solar wind speed's components in the corotating heliocentric spherical coordi-

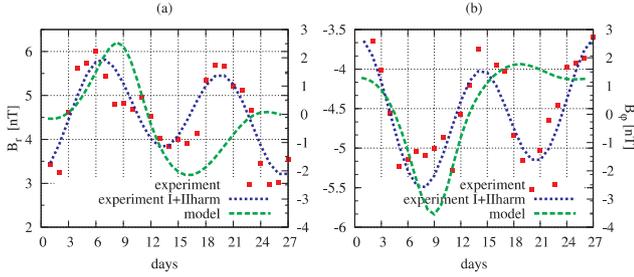


Fig. 3. Azimuthal changes of the superimposed by means of 13 BR (a) B_r and (b) B_ϕ components of the IMF at the Earth orbit; observations (red squares), observations approximated by the sum of the first and second harmonic waves (dotted lines) and values expected from Maxwell's equations (dashed lines) for the solar wind speed given by Eq. (1).

nate system (r, θ, φ) , as:

$$\begin{cases} \sin\theta V_r \frac{\partial B_\theta}{\partial\theta} + \sin\theta B_\theta \frac{\partial V_r}{\partial\theta} + \cos\theta V_r B_\theta \\ -V_\varphi \frac{\partial B_r}{\partial\varphi} - B_r \frac{\partial V_\varphi}{\partial\varphi} + V_r \frac{\partial B_\varphi}{\partial\varphi} + B_\varphi \frac{\partial V_r}{\partial\varphi} = 0 \\ V_\varphi \frac{\partial B_\theta}{\partial\varphi} + B_\theta \frac{\partial V_\varphi}{\partial\varphi} + r \sin\theta V_r \frac{\partial B_\theta}{\partial r} \\ + r \sin\theta B_\theta \frac{\partial V_r}{\partial r} + \sin\theta V_r B_\theta = 0 \\ r B_r \frac{\partial V_\varphi}{\partial r} + r V_\varphi \frac{\partial B_r}{\partial r} + V_\varphi B_r - V_r B_\varphi \\ -r V_r \frac{\partial B_\varphi}{\partial r} - r B_\varphi \frac{\partial V_r}{\partial r} + B_\theta \frac{\partial V_\varphi}{\partial\theta} + V_\varphi \frac{\partial B_\theta}{\partial\theta} = 0 \\ \frac{\partial B_r}{\partial r} + \frac{2}{r} B_r + \frac{c \sin\theta}{r} B_\theta + \frac{1}{r} \frac{\partial B_\theta}{\partial\theta} + \frac{1}{r \sin\theta} \frac{\partial B_\varphi}{\partial\varphi} = 0 \end{cases} \quad (3)$$

The latitudinal component B_θ of the IMF is very weak for the analyzed period, therefore further in this paper we consider 2-D model of the IMF ($B_\theta = 0$). These assumptions straightforwardly lead (from first equation in Eq. (3)) to the relationship between B_φ and B_r as, $B_\varphi = B_r \frac{V_\varphi}{V_r}$. Then last equation in Eq. (3) with respect to the radial component B_r of the IMF has a form:

$$A_1 \frac{\partial B_r}{\partial r} + A_2 \frac{\partial B_r}{\partial\varphi} + A_3 B_r = 0 \quad (4)$$

We take into account, as well $V_\theta = 0$, $V_\varphi = -\Omega r \sin\theta$ where V_φ is the corotational speed and Ω is the angular velocity of the Sun. We solve Eq. (4) by the numerical method described in detail in (Alania et al., 2009, 2010) with the boundary condition near the Sun $B_r[1, j, k] = 3770nT$ for $0^\circ < \theta \leq 90^\circ$ and $-3770nT$ for $90^\circ < \theta \leq 180^\circ$ for the positive polarity period ($A > 0$), where $i = 1, 2, \dots, I$; $j = 1, 2, \dots, J$; $k = 1, 2, \dots, K$ are steps in radial distance, vs. heliolatitude and heliolongitude, respectively. In considered case $r_1 = 0.1$ AU. The choice of these boundary conditions was stipulated by requiring agreement of the solutions of Eq. (4) with the in situ measurements of the B_r and B_φ components of the IMF at the Earth orbit. Presented in Fig. 3 are results of the solution of Eq. (4) for the B_r and B_φ components of the IMF. Figure 3 demonstrates that the expected B_r and B_φ components of the IMF differ from experimental data. The expected B_r and B_φ components preferentially contain the first harmonic of the 27-day variation, while experimental data clearly shows an

existence of the stronger second harmonic almost exceeding the first harmonic. Solutions of the Eqs. (2) as a self-consistent system must show at least a good correspondence between the changes of the solar wind velocity and IMF. We implemented in Eq. (4) in situ observed solar wind velocity consisting generally of the first harmonic, with the twice greater amplitude than the second harmonic (about 15% to 7% of the total solar wind velocity). So, it is clear that expected heliolongitudinal changes of B_r and B_φ components, reflecting a character of the changes of the implemented in situ observed solar wind velocity, do not contain noticeable second harmonics of the B_r and B_φ components. Consequently there is observed a distinction between observed and expected IMF.

2.2 Numerical solution of Parker's transport equation

Parker's transport equation was solved numerically as in papers published elsewhere (see e.g. Wawrzynczak and Alania, 2008). The parallel diffusion coefficient K_\parallel changes versus the spatial spherical coordinates (r, θ, φ) and rigidity R of GCR particles as, $K_\parallel = K_0 K(r) K(R)$, where $K_0 = \frac{\lambda_0 v}{3} \approx 10^{23} \text{ cm}^2/\text{s}$, v - is the velocity of GCR particles, and λ_0 - the transport free path of GCR particles; $K(r) = 1 + 0.5r$, r is in AU; $K(R) = (R/1 \text{ GV})^{0.5}$. So, the parallel diffusion coefficient K_\parallel for the GCR particles of 10 GV rigidity equals, $K_\parallel = 5 \times 10^{23} \text{ cm}^2 \text{ s}^{-1}$ at the Earth orbit. The ratios β and β_1 of the perpendicular K_\perp and drift K_d diffusion coefficients to the parallel diffusion coefficient K_\parallel of the GCR particles are given in standard form $\beta = \frac{K_\perp}{K_\parallel} = \frac{1}{1 + \omega^2 \tau^2}$ and $\beta_1 = \frac{K_d}{K_\parallel} = \frac{\omega \tau}{1 + \omega^2 \tau^2}$ where $\omega \tau = 300 B \lambda c R^{-1}$, c - speed of light, B - the strength of the IMF. The billiard ball diffusion is not generally the best approximation (Parhi et al., 2004; Shalchi, 2009) but it works well at high rigidities to which neutron monitor and muon telescopes are responsive, $R > 10 - 15 \text{ GV}$ (Jokipii, 1971; Shalchi, 2009). We included in Parker's transport equation the B_r and B_φ components and the magnitude $B = \sqrt{B_r^2 + B_\varphi^2}$ of the IMF obtained from the numerical solution of Eq. (4) with a variable solar wind speed (Eq. (1)). Implementation of the heliospheric magnetic field obtained from the numerical solution of Eq. (4) in Parker's transport equation is done through the spiral angle $\psi = \arctan(-\frac{B_\varphi}{B_r})$ in anisotropic diffusion tensor of GCR particles and ratios β and β_1 . The kinematical model of the IMF with variable solar wind speed has some limitations, especially it could be applied until some radius, while at large radii the fast wind would overtake the previously emitted slower one. So, we assume that an interaction between slow and fast streams of the solar wind velocity takes place not earlier than $\sim 8 \text{ AU}$. We justify an ignorance of corotating interaction regions by the absence of a noticeable Forbush decreases, and negligible correlation between changes of solar wind velocity and the magnitude of the IMF (correlation

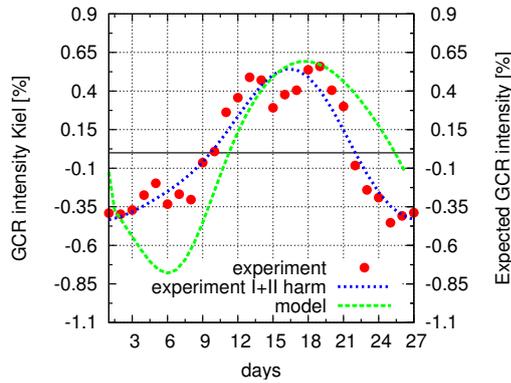


Fig. 4. Heliolongitudinal changes of the expected GCR intensity for effective rigidity 15 GV at the Earth orbit during one solar rotation period for the solar wind velocity assumed as in Eq. (1) (dashed line), temporal changes of the averaged GCR intensity by Kiel neutron monitor by means of 13 BR for the period of 12 January–28 December 1995 (points) and the approximation by the sum of the first and second harmonic waves of the observed GCR intensity (dotted line).

coefficient equals ~ 0.1) during the analyzed period. Thus, to exclude an intersection of the IMF lines the heliolongitudinal asymmetry of the solar wind speed takes place only up to the distance of ~ 8 AU and then $V = 400$ km/s and standard Parker's field is used throughout the heliosphere. The solution of the model of the 27-day variation of the GCR intensity corresponding to the $A > 0$ period (1995) are presented in Fig. 4 (dashed line). Figure 4 also presents average data of Kiel neutron monitor for 13 BR (2205–2217). We recalculated an experimental data to free space (beyond Earth's atmosphere and magnetosphere) for the power law type rigidity R spectrum $\delta D(R)/D(R) \propto R^{-\gamma}$ for $\gamma = 1.0$ (acceptable for the 27-day variations in the minimum epoch of solar activity, Gil and Alania, 2010) taking into account coupling coefficients and rigidity dependence of the amplitudes of the 27-day variation of the GCR intensity. Figure 4 shows that the range of the changes of the sum of the first and second harmonics of the 27-day variation of the GCR intensity for Kiel neutron monitor is little less than expected from the modelling. We must underline that generally there is not any problem to adjust modelling results to the experimental data by alternating values of free parameters. We believe that the modelling results for 15 GV rigidity particles of GCR can be successfully compared with Kiel neutron monitor data corresponding to the effective energy of $\sim 10 - 15$ GeV.

3 Discussion and Conclusions

We constructed a 3-D model of the 27-day variation of the GCR intensity with implementing in the average solar wind velocity for the BR (2205–2217) in period of 12 January–28 December of 1995 ($A > 0$), possessing properties as follows: (a) the regular part V_0 of the solar wind velocity

changes versus heliolatitudes in accord to in situ measurements of the Ulysses spacecraft; (b) the heliolongitudinal asymmetry of solar wind velocity reproducing as the sum of the first and second harmonics of the 27-day variation depends on the heliolatitudes. We implement in a model B_r and B_ϕ components of the IMF derived from the Maxwell's equations corresponding to the changeable solar wind velocity (Eq. (1)). The average value of the B_θ component is negligible for the considered period. We assume that an interaction between slow and fast streams of the solar wind velocity takes place not earlier than ~ 8 AU. We justify an ignorance of corotating interaction regions by the absence of a noticeable Forbush decreases, and negligible correlation between changes of solar wind velocity and the magnitude of the IMF (correlation coefficient equals ~ 0.1) during the analyzed period. We show that, though, the range of changes of the sum of the first and second harmonics of the 27-day variation of the GCR intensity for Kiel neutron monitor is little less than expected from the modelling, however, they are comparable. We believe that a comparison of the modelling results for 15 GV rigidity particles of GCR can be effectively compared with Kiel neutron monitor data corresponding to the effective energy of $\sim 10 - 15$ GeV.

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