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An evaluation of electron-photon cascades developing in matter, photon and magnetic fields

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Abstract. Electromagnetic cascades in matter, photon fields, and magnetic fields are solved by a standard numerical method, integrating the diffusion equations of respective cascade processes numerically. Our results and those of Aharonian and Plyasheshnikov agree very well for cascades in matter and magnetic fields, though they show some slight discrepancies for cascades in photon fields of high incident energies. Transport properties of electron and photon spectra are also investigated by solving differential-difference equations for cascades with simplified cross-sections, and the spectra under the electron cooldown process are well explained quantitatively.

1 Introduction

Investigations of electron-photon cascades developing in matter, photon fields, and magnetic fields are very important for astrophysical studies in the high-energy gamma-ray astronomy, as Aharonian and Plyasheshnikov (2003) pointed out strongly in results derived from the adjoint method by Plyasheshnikov et al. (1979). Here we introduce another approach to solve cascades developing in those environments for mutual confirmations among results and solving methods, in order to improve the accuracies and simplifications of computations, and for future applications in other or more complicated environments.

The diffusion equation for cascades in matter is reviewed in Rossi and Greisen (1941), and that in magnetic fields was described in Akhiezer et al. (1994). In this report, we propose the diffusion equation of cascades in photon fields. It is difficult to solve integro-differential equations for cascades by the traditional analytical method shown in, for example, Rossi and Greisen, when cross-sections are inhomogeneous with primary and secondary energies. So we apply standard numerical methods to integrate the above cascade equations.

Our results for transition curves of shower electrons developing in matter, photon fields, and magnetic fields are compared with those derived by Aharonian and Plyasheshnikov and by Nishimura (1967). We also indicate changes in the energy spectra of cascade electrons and photons penetrating photon fields, and attempt to explain the transport property of the energy spectra quantitatively by solving differential-difference equations with simplified cross-sections.

2 Cascades developing in matter, photon fields, and magnetic fields

2.1 Diffusion equation for the cascades in photon fields

Let $\pi(\kappa,t)d\kappa$ and $\gamma(\lambda,t)d\lambda$ be the differential energy spectra of shower electrons and photons with

$$\kappa \equiv \omega_0 \varepsilon_{\rm e} \qquad \text{and} \qquad \lambda \equiv \omega_0 \varepsilon_\gamma, \tag{1}$$

where $\varepsilon_{\rm e}$, ε_γ, and ω_0 denote the energies of the shower electron, shower photon, and background photon in units of mc^2, respectively, after penetrating mono-energetic and isotropic photon fields of depth t in units of

$$X_0^{(\rm G)} = \left[4\pi n_0^{(\rm G)} r_0^2\right]^{-1} \kappa_0, \tag{2}$$

defined in Aharonian and Plyasheshnikov (2003), where κ_0 denotes κ or λ of the incident particle.

Then the diffusion equation for the electron-photon cascades in the photon fields can be described as

$$\frac{\partial \pi(\kappa,t)}{\kappa_0 \partial t} = -\frac{\pi(\kappa)}{\kappa}\int_0^1 \phi(\kappa,v)dv + \int_\kappa^{\kappa_0} \phi(\kappa',1-\frac{\kappa}{\kappa'})\frac{\pi(\kappa')}{\kappa'}\frac{d\kappa'}{\kappa'} + 2\int_\kappa^{\kappa_0} \psi(\lambda',\frac{\kappa}{\lambda'})\frac{\gamma(\lambda')}{\lambda'}\frac{d\lambda'}{\lambda'}, \tag{3}$$

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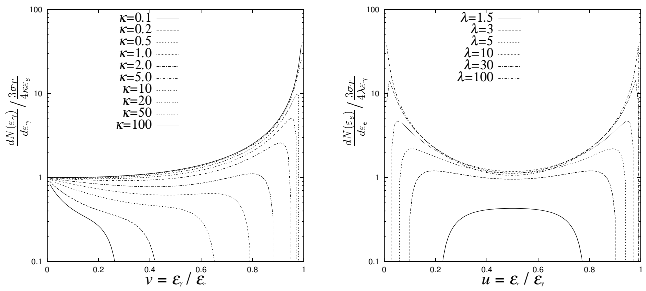

Fig. 1. The normalized cross-sections for the inverse Compton scattering $\frac{dN_{\mathrm{IC}}(\varepsilon_\gamma)}{d\varepsilon_\gamma}/\frac{3\sigma_{\mathrm{T}}}{4\kappa\varepsilon_e}$, where $\kappa=.1,.2,.5,\cdots,100$, from bottom to top (left), and the photon-photon pair production $\frac{dN_{\mathrm{PP}}(\varepsilon_e)}{d\varepsilon_e}/\frac{3\sigma_{\mathrm{T}}}{4\lambda\varepsilon_\gamma}$, where $\lambda=1.5,3,5,10,30,100$, from bottom to top (right).

$$\frac{\partial\gamma(\lambda,t)}{\kappa_0\partial t}=\int_\lambda^{\kappa_0}\phi(\kappa',\frac{\lambda}{\kappa'})\frac{\pi(\kappa')}{\kappa'}\frac{d\kappa'}{\kappa'}-\frac{\gamma(\lambda)}{\lambda}\int_0^1\psi(\lambda,u)du, \quad (4)$$

where $u\equiv\varepsilon_{\mathrm{e}}/\varepsilon_\gamma$ and $v\equiv\varepsilon_\gamma/\varepsilon_{\mathrm{e}}$ denote fractional energies and $\frac{dN_{\mathrm{IC}}(\varepsilon_\gamma)}{d\varepsilon_\gamma}\equiv\frac{3\sigma_{\mathrm{T}}}{2\kappa\varepsilon_e}\phi(\kappa,v)$ and $\frac{dN_{\mathrm{PP}}(\varepsilon_e)}{d\varepsilon_e}\equiv\frac{3\sigma_{\mathrm{T}}}{2\lambda\varepsilon_\gamma}\psi(\lambda,u)$ denote the cross-sections indicated in Fig. 1 for the inverse Compton scattering and the photon-photon pair production, respectively, as described in Aharonian (2004) and Zdziarski (1988), with σ_{T} of the Thomson cross-section. They are

$$\begin{aligned}\phi(\kappa,v)=&\frac{1}{4}\left(1-v+\frac{1}{1-v}\right)\\&+\frac{v}{16\kappa(1-v)}\left(3+v-\frac{1}{1-v}+4\ln\frac{v}{4\kappa(1-v)}\right)\\&-\frac{v^2}{16\kappa^2(1-v)^2}\quad\text{with}\quad v<1/\left(1+\frac{1}{4\kappa}\right),\end{aligned}\quad (5)$$

$$\begin{aligned}\psi(\lambda,u)=&\frac{1}{4}\left(\frac{1-u}{u}+\frac{u}{1-u}\right)\\&-\frac{1}{16\lambda u(1-u)}\left(4+\frac{1-u}{u}+\frac{u}{1-u}-4\ln\{4\lambda u(1-u)\}\right)\\&+\frac{1}{16\lambda^2u^2(1-u)^2}\quad\text{with}\\&\frac{1}{2}\left(1-\sqrt{1-\frac{1}{\lambda}}\right)<u<\frac{1}{2}\left(1+\sqrt{1-\frac{1}{\lambda}}\right).\end{aligned}\quad (6)$$

2.2 Results obtained by standard numerical integrations

We derived the transition curves of electron-photon cascades developing in matter, photon fields, and magnetic fields in Fig. 2 by numerically solving the diffusion equations indicated in Rossi and Greisen (1941), the preceding subsection, and Akhiezer et al. (1994), respectively.

We applied the trapezoidal method for integrations with energy on a logarithmic scale. The results of the cascades in matter agree very well with those indicated in Nishimura (1967), and also the results of cascades in magnetic fields agree very well with those of Aharonian and Plyasheshnikov (2003). Nevertheless, we could not get very good agreement for cascades in photon fields with the incident energy fixed to $\kappa_0=10^3$, with those obtained by Aharonian and Plyasheshnikov, as compared in Fig. 2.

It was very difficult to carry out accurate integrations with energy in Eqs. (3) and (4) in cases of high primary energies, due to the appearance of a very sharp peak in the cross-sections of Fig. 1 at fractional energies (v and u) of almost 1. We therefore applied variable transformations of the double exponential type in those integrations, and thus obtained qualitatively good results as shown in Fig. 3 for shower curves with the threshold energy fixed to $\kappa_{\mathrm{thr}}=1$. Quantitative comparisons with the results of Aharonian and Plyasheshnikov in Fig. 3 show good agreement for the curve with the incident energy of $\kappa_0=10$. However, our curves with the higher incident energies differ significantly from theirs in peak electron numbers and peak positions. One probable reason for such large discrepancies is that Aharonian and Plyasheshnikov might use a doubled value for the radiation length $X_0^{(\mathrm{G})}$ of Eq. (2) except for the case of $\kappa_0=10$ in Fig. 3, though further investigations are necessary to get consistent results.

3 Transport property of energy spectra for cascades in photon fields

The energy spectra of cascade electrons and photons show characteristic shapes and changes in photon fields, as indicated in Figs. 4 and 5. We attempt to explain the transport properties of electron and photon spectra by analytically solving differential-difference equations for cascades in photon fields with simplified cross-sections. To learn the reasons for these characteristic features of the transition of this cascade, we use the analytical solution of the cascade in the photon fields with cross-sections simplified from Eqs. (5) and (6).

3.1 Generation-separated differential energy spectra

We assume

$$\phi(\kappa,v)=\frac{1}{2},\qquad\text{and}\qquad\psi(\lambda,u)=\frac{1}{2}, \quad (7)$$

or simply approximate the cross-sections to be[1]

$$\frac{dN_{\mathrm{IC}}(\varepsilon_\gamma)}{d\varepsilon_\gamma}\simeq\frac{3\sigma_{\mathrm{T}}}{4\kappa\varepsilon_e},\qquad\text{and}\qquad\frac{dN_{\mathrm{PP}}(\varepsilon_e)}{d\varepsilon_e}\simeq\frac{3\sigma_{\mathrm{T}}}{4\lambda\varepsilon_\gamma}, \quad (8)$$

[1] If the value of $X_0^{(\mathrm{G})}$ is doubled, the values of $\phi(\kappa,v)$ and $\psi(\lambda,u)$ are doubled. Then ϕ and ψ agree with Fig. 1 and act as the correction factors of the approximated cross-sections (8), when expressions appearing in this subsection are slightly simplified as $t/2$ to be replaced by t.

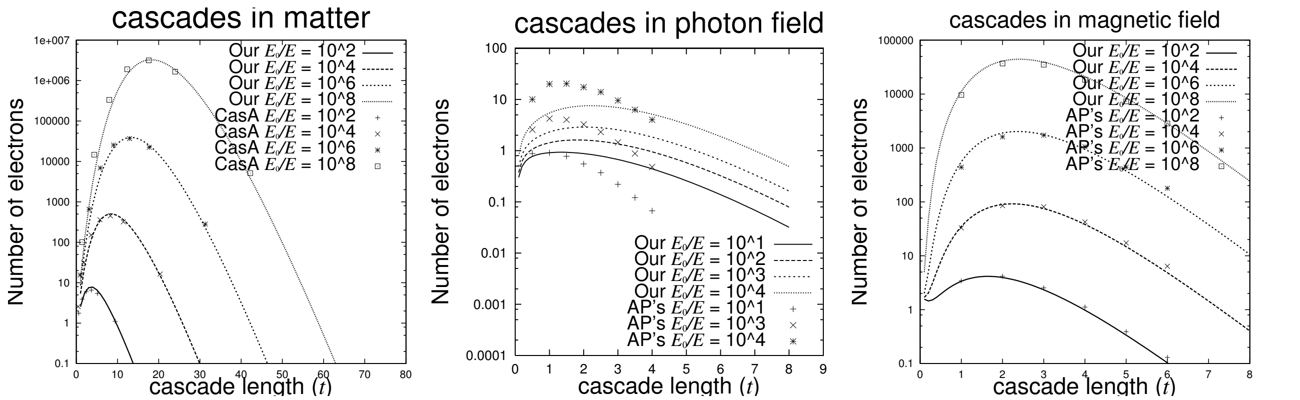

Fig. 2. Transition curves of shower electron developing in matter (left), photon fields (middle), and magnetic fields (right). Our results in matter (lines) are compared with the analytical results in Nishimura (dots), and our results in photon fields with $\kappa_0 = 10^3$ and in magnetic fields with cascade length defined by $3.9\times 10^6 (\frac{H}{H_c})^{\frac{2}{3}} \frac{x}{cm}/(\frac{E_0}{GeV})^{\frac{1}{3}}$ (lines) are compared with those of Aharonian and Plyasheshnikov (dots).

and attempt to reproduce the shower spectra indicated in Figs. 4 and 5. Then the diffusion Eqs. (3) and (4) are described as

$$\frac{\partial}{\partial t}\pi(\kappa,t) + \frac{\kappa_0}{2\kappa}\pi(\kappa,t) - \frac{\kappa_0}{2}\int_{\kappa}^{\kappa_0}\frac{\pi(\kappa',t)}{\kappa'^2}d\kappa' = \kappa_0\int_{\kappa}^{\kappa_0}\frac{\gamma(\lambda',t)}{\lambda'^2}d\lambda', \quad (9)$$

$$\frac{\partial}{\partial t}\gamma(\lambda,t) + \frac{\kappa_0}{2\lambda}\gamma(\lambda,t) = \frac{\kappa_0}{2}\int_{\lambda}^{\kappa_0}\frac{\pi(\kappa',t)}{\kappa'^2}d\kappa'. \quad (10)$$

At first, we derive the Green functions for electron and photon transports, $G_\pi(\kappa,t;\kappa')$ and $G_\gamma(\lambda,t;\lambda')$, by replacing right hand sides of Eqs. (9) and (10) with $\delta(\kappa-\kappa')$ and $\delta(\lambda-\lambda')$, respectively. Applying Mellin transforms

$$\mathcal{M}(s,t) = \int_0^\infty \left(\frac{\kappa}{\kappa_0}\right)^s \pi(\kappa,t)d\kappa, \quad (11)$$

$$\mathcal{N}(s,t) = \int_0^\infty \left(\frac{\lambda}{\kappa_0}\right)^s \gamma(\lambda,t)d\lambda, \quad (12)$$

we get differential-difference equations

$$\frac{\partial}{\partial t}\mathcal{M}(s,t) + \frac{s}{2(s+1)}\mathcal{M}(s-1,t) = \left(\frac{\kappa'}{\kappa_0}\right)^s, \quad (13)$$

$$\frac{\partial}{\partial t}\mathcal{N}(s,t) + \frac{1}{2}\mathcal{N}(s-1,t) = \left(\frac{\lambda'}{\kappa_0}\right)^s. \quad (14)$$

The solutions for respective Mellin transforms are

$$\mathcal{M}(s,t) = (\kappa'/\kappa_0)^s\left(1+\frac{\kappa_0 t}{2\kappa'(s+1)}\right)e^{-\kappa_0 t/(2\kappa')}, \quad (15)$$

$$\mathcal{N}(s,t) = (\lambda'/\kappa_0)^s e^{-\kappa_0 t/(2\lambda')}, \quad (16)$$

and applying inverse Mellin transforms, we have

$$G_\pi(\kappa,t;\kappa') = \left\{\delta(\kappa-\kappa') + \frac{\kappa_0 t}{2\kappa'^2}\right\}e^{-\kappa_0 t/(2\kappa')}, \quad (17)$$

$$G_\gamma(\lambda,t;\lambda') = \delta(\lambda-\lambda')e^{-\kappa_0 t/(2\lambda')}. \quad (18)$$

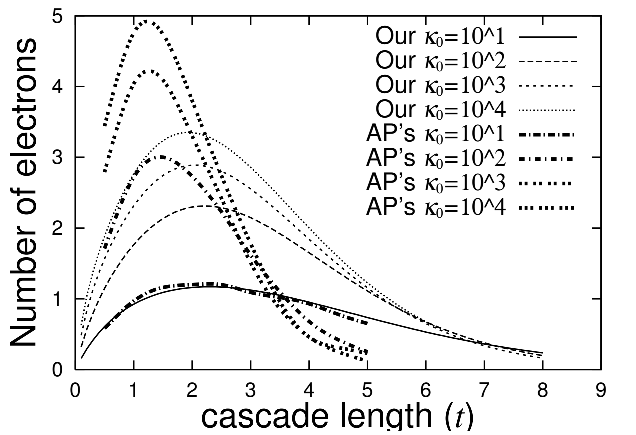

Fig. 3. Comparison of transition curves of shower electron in photon fields between ours (thin lines) and Aharonian and Plyasheshnikov's (thick lines), with the threshold energy of $\kappa_{\rm thr} = 1$, and with the incident photon energies of $\kappa_0 = 10$, 10^2, 10^3, and 10^4, from bottom to top.

Applying the Green functions to Eqs. (9) and (10), we get the differential spectra up to the second generation on which suffix 1 corresponds to the first generation and 2 to the second generation:

$$\gamma_1(\lambda,t) = G_\gamma(\lambda,t;\kappa_0) = \delta(\lambda-\kappa_0)e^{-t/2}, \quad (19)$$

$$\pi_1(\kappa,t) = \int_0^t dt'\frac{e^{-t'/2}}{\kappa_0}\int_\kappa^{\kappa_0} G_\pi(\kappa,t-t';\kappa')d\kappa' = \frac{t}{\kappa_0}e^{-t/2}, \quad (20)$$

$$\gamma_2(\lambda,t) = \int_0^t dt'\frac{t'}{2}e^{-t'/2}\int_\lambda^{\kappa_0}\left(\frac{1}{\lambda'}-\frac{1}{\kappa_0}\right)G_\gamma(\lambda,t-t';\lambda')d\lambda' = \frac{1}{\kappa_0}\left\{te^{-t/2} - \frac{2u}{1-u}\left(e^{-t/2}-e^{-t/(2u)}\right)\right\}, \quad (21)$$

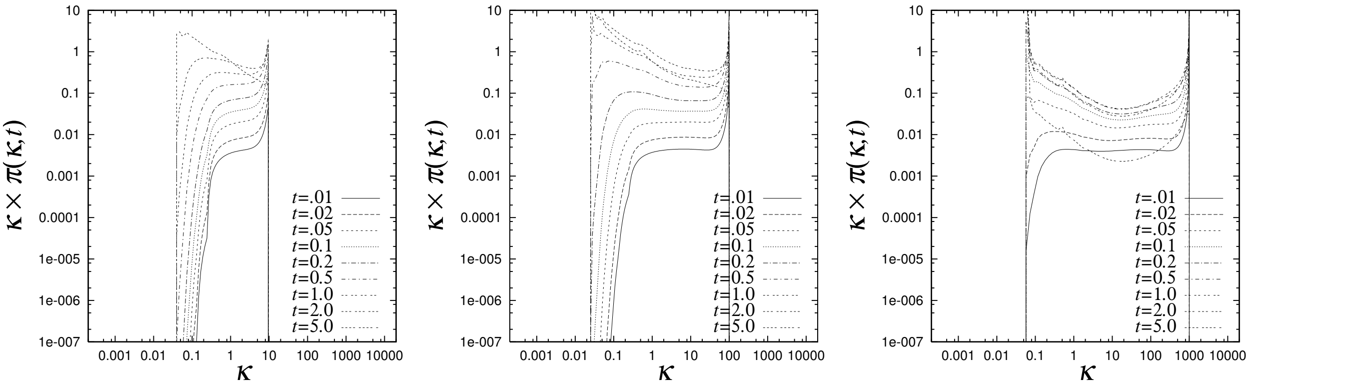

Fig. 4. κ-weighted differential energy spectra of electron, $\kappa\pi(\kappa,t)$, at depths of $t=.01,.02,.05,.1,.2,.5,1,2,5$, for incident photon of $\kappa_0=10$, 10^2, and 10^3, from left to right.

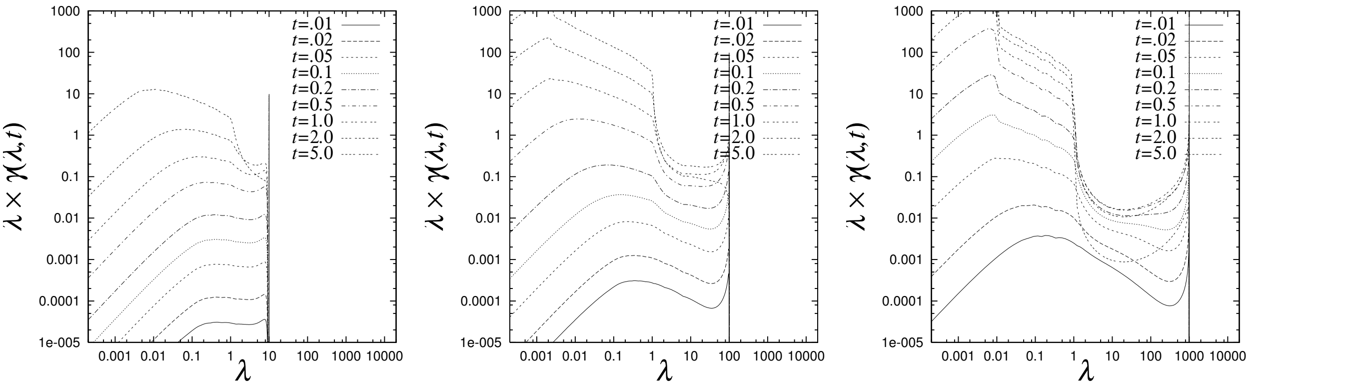

Fig. 5. λ-weighted differential energy spectra of photons, $\lambda\gamma(\lambda,t)$, at depths of $t=.01,.02,.05,.1,.2,.5,1,2,5$, for incident photons of $\kappa_0=10$, 10^2, and 10^3, from left to right.

$$\begin{aligned}\pi_2(\kappa,t) &= \int_0^t dt' \int_\kappa^{\kappa_0} d\mu G_\pi(\kappa,t-t';\mu) \\ &\quad \times \int_\mu^{\kappa_0} \frac{1}{\kappa'^2}\left\{t' e^{-t'/2} - \frac{2\kappa'}{\kappa_0-\kappa'}\left[e^{-t'/2} - e^{-\frac{t'}{2\kappa'/\kappa_0}}\right]\right\} d\kappa' \\ &= \frac{2}{\kappa_0}\int_0^{1-v} \frac{1}{v'(1-v')}\left[t\left(e^{-t/2/(1-v')} + e^{-t/2}\right)\right. \\ &\quad \left. + \frac{4(1-v')}{v'}\left(e^{-t/2/(1-v')} - e^{-t/2}\right)\right] dv', \qquad (22)\end{aligned}$$

where $\gamma_1(\lambda,t)$ is the spectrum of the incident photon and its survival, $\pi_1(\kappa,t)$ that of electrons produced by γ_1, $\gamma_2(\lambda,t)$ that of photons produced by π_1, and $\pi_2(\kappa,t)$ that of electrons produced by γ_2; v and u denote the fractional energies κ/κ_0 and λ/κ_0, respectively. The κ and λ weighted differential spectra $\kappa\pi_1(\kappa,t)$, $\lambda\gamma_2(\lambda,t)$, and $\kappa\pi_2(\kappa,t)$ are functions of v and u, as indicated in Fig. 6.

$\pi_1(\kappa,t)$ does not depend on κ and increases proportionally to t at $t \ll 1$, which well explains the early stage ($t<1$) of the electron spectrum of $\kappa_0=10$ in Fig. 4. $\gamma_2(\lambda,t)$ is approximated by $\frac{1}{4}\left(\frac{1}{\lambda}-\frac{1}{\kappa_0}\right)t^2$ at $t \ll 1$, which well explains the photon spectrum of $\kappa_0=10$ at $\lambda>0.1$ in Fig. 5.

We could explain the above few stages but could only slightly explain other stages of shower spectra by our generation-separated energy spectra, Eqs. (19)–(22). This was because the approximated cross-sections (8) were too simple, valid only for κ and λ between 1 and 10. More accurate cross-sections should be taken into account to reproduce the spectra of showers initiated by higher incident energies.

3.2 Electron cooldown process

For the particle energies of $\kappa<1$, the pair production process is suppressed by the inequality in Eq. (6). As a result, photons stop interacting in the field and in turn electrons are not produced from photons, and electrons only lose their energies by the inverse Compton scattering. We call this process the electron cooldown process.

At energy regions of $\kappa \ll 1$, the inverse Compton cross-section is approximated with a considerable accuracy by

$$\phi(\kappa,v) \simeq \frac{1}{2}, \qquad \text{with} \qquad v < 1\Big/\left(1+\frac{1}{4\kappa}\right) \simeq 4\kappa, \qquad (23)$$

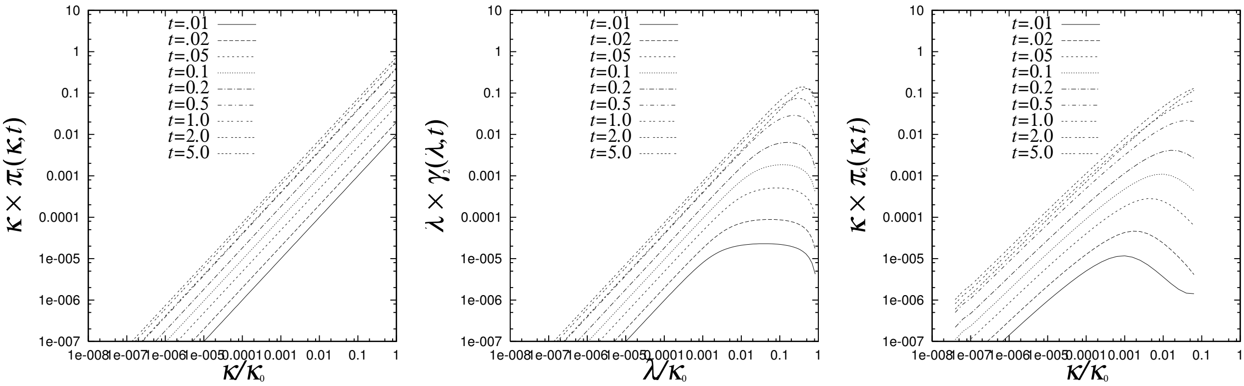

Fig. 6. The generation-separated differential energy spectra (κ or λ weighted). The electron spectrum $\pi_1(\kappa,t)$ produced by the survival of the incident photon of $\delta(\lambda-\kappa_0)$, the photon spectrum $\gamma_2(\lambda,t)$ produced by $\pi_1(\kappa,t)$, and the electron spectrum $\pi_2(\kappa,t)$ produced by $\gamma_2(\lambda,t)$, from left to right.

then electrons diffuse as

$$\begin{aligned}\frac{\partial \pi(\kappa,t)}{\kappa_0 \partial t} &= \frac{1}{\kappa}\int_0^1 \left\{\phi(\frac{\kappa}{1-v},v)\pi(\frac{\kappa}{1-v},t)-\phi(\kappa,v)\pi(\kappa,t)\right\}dv \\ &\simeq \int_0^1 v\frac{\partial}{\partial \kappa}\{\phi(\kappa,v)\pi(\kappa,t)\}dv \\ &\simeq 8\kappa\pi(\kappa,t)+4\kappa^2\frac{\partial}{\partial\kappa}\pi(\kappa,t). \qquad (24)\end{aligned}$$

Applying Mellin transforms, we have

$$\frac{\partial}{\partial t}\mathcal{M}(s,t)+4\kappa_0^2 s\mathcal{M}(s+1,t)=0. \qquad (25)$$

The solution for the initial condition $\pi(\kappa,0)=\delta(\kappa-\kappa')$ is

$$\mathcal{M}(s,t)=\left(\frac{\kappa_0}{\kappa'}+4\kappa_0^2 t\right)^{-s}, \qquad (26)$$

then, applying inverse Mellin transforms, we have

$$\begin{aligned}\pi(\kappa,t) &= \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}\frac{\kappa_0{}^s}{\kappa^{s+1}}\left(\frac{\kappa'}{\kappa_0+4\kappa_0^2 t\kappa'}\right)^s ds \\ &= \delta(\kappa-(\frac{1}{\kappa'}+4\kappa_0 t)^{-1}). \qquad (27)\end{aligned}$$

This solution indicates that the electrons lose their energy from κ' to κ as

$$\frac{1}{\kappa}=\frac{1}{\kappa'}+4\kappa_0 t, \qquad (28)$$

after the penetration of depth t.[2] As electrons produced by the pair production process have some finite energies due to the inequality in Eq. (6), electron spectra have the lower cut-off κ_{lc}. The value of κ_{lc} decreases according to Eq. (28), as typically shown in Fig. 4 of $\kappa_0=10^2$.

[2] This formula is also derived from the equation of the mean energy dissipation, $-\frac{d}{\kappa_0 dt}\kappa=\int_0^1 v\phi(\kappa,v)dv \simeq \frac{1}{2}\int_0^{4\kappa} v dv = 4\kappa^2$.

Under the cascade process in the photon field, electrons are produced from photons by a spectrum form of $\pi(\kappa',t)d\kappa' \simeq \mathrm{const.}\ d\kappa'$, according to the pair production cross-section in Fig. 1. After the cooling down, they show the spectrum form of

$$\begin{aligned}\pi(\kappa,t)d\kappa &= \mathrm{const.}\ \times\ \frac{\partial\kappa'}{\partial\kappa}d\kappa \\ &= \mathrm{const.}\ \times\ (1-4\kappa_0\kappa t)^{-2}d\kappa \\ &\simeq \frac{\mathrm{const.}}{16\kappa_0^2 t^2}\kappa^{-2}d\kappa \quad \text{for} \quad 4\kappa_0\kappa t \gg 1. \qquad (29)\end{aligned}$$

Thus electrons show power-law type energy spectra of index -2, as indicated in Fig. 4 in energy regions of $\kappa<1$ at $t>1$ for $\kappa_0=10^2$ and at $t>0.1$ for $\kappa_0=10^3$.

Under the electron cooldown process, photons of energy λ are supplied only by electrons of energy greater than κ' of

$$\kappa'=(\lambda+\sqrt{\lambda^2+\lambda})/2 \simeq \sqrt{\lambda}/2, \qquad (30)$$

according to the inequality of Eq. (23). They then stop interacting in the field, so the diffusion Eq. (10) becomes

$$\frac{\partial}{\kappa_0\partial t}\gamma(\lambda,t)=\int_{\sqrt{\lambda}/2}^{\kappa_0}\frac{\pi(\kappa',t)}{2\kappa'}\frac{d\kappa'}{\kappa'}. \qquad (31)$$

We assume the power-law type electron spectrum, $\pi(\kappa,t)\simeq\kappa^{-2}f(t)$, according to Eq. (29). As electrons cool down very slowly, their energy spectra have the lower cut-off κ_{lc}. When the lower bound of the integral in Eq. (31) is contained in the electron spectrum, or $\sqrt{\lambda}/2>\kappa_{\mathrm{lc}}$, Eq. (31) becomes

$$\frac{\partial}{\kappa_0\partial t}\gamma(\lambda,t)\simeq\frac{f(t)}{6}\left(\frac{\sqrt{\lambda}}{2}\right)^{-3}\simeq\frac{4}{3}f(t)\lambda^{-3/2}. \qquad (32)$$

So photons show power-law type energy spectra of index $-3/2$, as typically indicated in Fig. 5 at $0.01<\kappa<1$, $t>0.5$ for $\kappa_0=10^2$ and at $0.01<\kappa<1$, $t>0.1$ for $\kappa_0=10^2$.

On the other hand, when the lower bound of the integral in Eq. (31) is out of the electron spectrum, or $\sqrt{\lambda}/2 < \kappa_{\rm lc}$, the photon spectrum $\gamma(\lambda, t)$ becomes constant irrespective of their energy λ, as typically indicated in Fig. 5 at about $\kappa < 0.01$ for $\kappa_0 = 10$, about $\kappa < 0.01$, $t < 0.5$ for $\kappa_0 = 10^2$ and about $\kappa < 0.01$, $t < 0.1$ for $\kappa_0 = 10^3$ for $\kappa_0 = 10^3$. These results quantitatively support the investigations of Aharonian and Plyasheshnikov (2003) for the energy spectra in these regions.

4 Conclusions

We evaluated electron-photon cascades developing in matter, photon fields, and magnetic fields by solving diffusion equations using a standard numerical integration method, and compared the results with the analytical results indicated in Nishimura (1967) and with those derived by the adjoint method of Aharonian and Plyasheshnikov (2003).

Our results for cascades in matter and in the magnetic fields agreed well with those indicated in Nishimura and Aharonian and Plyasheshnikov, though those in photon fields showed a slight discrepancy for showers with high incident energies ($\kappa_0 \geq 100$), where the cross-sections increase very steeply near the primary energy and the computation times are increased.

We also investigated the transport property of energy spectra of cascades in photon fields by obtaining the analytical solutions of differential-difference equations assuming simplified cross-sections. We partially succeeded in explaining electron and photon spectra of $\kappa > 0.1$ at very early stages ($t < 0.1$) for cascades with low incident energy ($\kappa_0 = 10$), though more accurate cross-sections than Eq. (8) should be taken into account for cascades with higher incident energies. Solutions of the electron cooldown process based on the cross-sections with Eq. (23) well explained the shapes of the electron and photon spectra of very low energy ($\kappa \ll 1$) quantitatively. We are improving our method to make it applicable to higher energy regions and deeper developing stages.

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